ESSENTIAL QUESTION

How can you use relationships in two variables to solve real-world problems?

You can use tables, graphs, and equations in two variables to model real-world problems, then use these representations to solve the problems.

Real-World Video

A two-variable equation can represent an animal’s distance over time. A graph can display the relationship between the variables. You can graph two or more animals’ data to visually compare them.
**Are You Ready?**

**Assess Readiness**

Use the assessment on this page to determine if students need intensive or strategic intervention for the module’s prerequisite skills.

**Intervention**

Access Are You Ready? assessment online, and receive instant scoring, feedback, and customized intervention or enrichment.

**Enrichment**

Skills Intervention worksheets
- Skill 36 Multiplication Facts
- Skill 69 Graph Ordered Pairs (First Quadrant)

Differentiated Instruction
- Challenge worksheets

**Online and Print Resources**

- Personal Math Trainer: Online Assessment and Intervention
- Online Teacher Edition
- ePlanner
- Interactive Answers and Solutions

**PROFESSIONAL DEVELOPMENT VIDEO**

Author Juli Dixon models successful teaching practices as she explores graphing in the coordinate plane in an actual sixth-grade classroom.

**Are YOU Ready?**

Complete these exercises to review skills you will need for this module.

**Multiplication Facts**

**EXAMPLE**

\[ 8 \times 7 = 56 \]

Use a related fact you know.

\[ 7 \times 7 = 49 \]

\[ \text{Think: } 8 \times 7 = (7 \times 7) + 7 \]

\[ = 49 + 7 \]

\[ = 56 \]

Multiply.

1. \[ 7 \times 6 = 42 \]
2. \[ 10 \times 9 = 90 \]
3. \[ 13 \times 12 = 156 \]
4. \[ 8 \times 9 = 72 \]

Write the rule for each table.

5. \[
\begin{array}{c|cccc}
\hline
x & 1 & 2 & 3 & 4 \\
y & 7 & 14 & 21 & 28 \\
\hline
\end{array}
\]

\( y \) is 7 times \( x \).

6. \[
\begin{array}{c|cccc}
\hline
x & 1 & 2 & 3 & 4 \\
y & 7 & 8 & 9 & 10 \\
\hline
\end{array}
\]

\( y \) is 6 more than \( x \).

7. \[
\begin{array}{c|cccc}
\hline
x & 1 & 2 & 3 & 4 \\
y & 5 & 10 & 15 & 20 \\
\hline
\end{array}
\]

\( y \) is 5 times \( x \).

8. \[
\begin{array}{c|cccc}
\hline
x & 0 & 4 & 8 & 12 \\
y & 0 & 2 & 4 & 6 \\
\hline
\end{array}
\]

\( y \) is one-half \( x \).

**Graph Ordered Pairs (First Quadrant)**

**EXAMPLE**

Start at the origin. Move 9 units right. Then move 5 units up. Graph point \( A(9, 5) \).

Graph each point on the coordinate grid above.

9. \( B(0, 8) \)
10. \( C(2, 3) \)
11. \( D(6, 7) \)
12. \( E(5, 0) \)

**Online Teacher Edition**

Access a full suite of teaching resources online—plan, present, and manage classes and assignments.

**Interactive Whiteboards**

Engage students with interactive whiteboard-ready lessons and activities.

**Interactive Answers and Solutions**

Customize answer keys to print or display in the classroom. Choose to include answers only or full solutions to all lesson exercises.

**Personal Math Trainer: Online Assessment and Intervention**

Assign automatically graded homework, quizzes, tests, and intervention activities. Prepare your students with updated practice tests aligned with Common Core.
Reading Start-Up

Have students complete the activities on this page by working alone or with others.

Visualize Vocabulary

The chart helps students review vocabulary associated with algebraic expressions. Write additional expressions on the board and have students identify the parts of each expression.

Understand Vocabulary

Use the following explanations to help students learn the preview words.

A coordinate plane is formed by two number lines that intersect at right angles. Coordinate planes are used in geographic maps and for locating images on computer screens.

The two lines that make a coordinate plane are called the axes. The x-axis is the horizontal number line that runs left to right on the coordinate plane. The y-axis is the vertical line that runs up and down on the coordinate plane.

Active Reading

Integrating Language Arts

Students can use these reading and note-taking strategies to help them organize and understand new concepts and vocabulary.

Additional Resources

Differentiated Instruction

• Reading Strategies [ELL]

**Visualize Vocabulary**

Use the ✔ words to complete the chart.

<table>
<thead>
<tr>
<th>Parts of the Algebraic Expression</th>
<th>Definition</th>
<th>Mathematical Representation</th>
<th>Review Word</th>
</tr>
</thead>
<tbody>
<tr>
<td>14 + 3x</td>
<td></td>
<td>14</td>
<td>constant</td>
</tr>
<tr>
<td>A number that is multiplied by a variable in an algebraic expression</td>
<td>3</td>
<td>coefficient</td>
<td></td>
</tr>
<tr>
<td>A letter or symbol used to represent an unknown</td>
<td>x</td>
<td>variable</td>
<td></td>
</tr>
</tbody>
</table>

**Understand Vocabulary**

Complete the sentences using the preview words.

1. The numbers in an ordered pair are ___coordinates___.
2. A ___coordinate plane___ is formed by two number lines that intersect at right angles.

**Active Reading**

Layered Book: Before beginning the module, create a layered book to help you learn the concepts in this module. Label each flap with lesson titles from this module. As you study each lesson, write important ideas such as vocabulary and formulas under the appropriate flap. Refer to your finished layered book as you work on exercises from this module.

**Before**

Students understand:
- how to recognize the difference between additive and multiplicative numerical patterns given in a table or graph
- how to graph a relationship on a number line
- how to identify and locate ordered pairs of whole numbers in the first quadrant

**In this module**

Students will learn to:
- identify independent and dependent quantities from tables and graphs
- write an equation that represents the relationship between independent and dependent quantities from a table
- represent a given situation using verbal descriptions, tables, graphs, and equations in the form \( y = kx \) or \( y = x + b \)
- graph points in all four quadrants using ordered pairs of rational numbers

**After**

Students will connect:
- tables and verbal descriptions with a linear relationship
- graphs and equations with a linear relationship
- ordered pairs with an equation
Unpacking the Standards

Use the examples on this page to help students know exactly what they are expected to learn in this module.

Common Core Standards

Content Areas

Expressions and Equations—6.EE

Represent and analyze quantitative relationships between dependent and independent variables.

Go online to see a complete unpacking of the Common Core Standards.

What It Means to You

You will learn to write an equation that represents the relationship in a table.

UNPACKING EXAMPLE 6.EE.9

Emily has a dog-walking service. She charges a daily fee of $7 to walk a dog twice a day. Create a table that shows how much Emily earns for walking 1, 6, 10, and 15 dogs. Write an equation that represents the situation.

<table>
<thead>
<tr>
<th>Dogs walked</th>
<th>1</th>
<th>6</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earnings ($)</td>
<td>7</td>
<td>42</td>
<td>70</td>
<td>105</td>
</tr>
</tbody>
</table>

Earnings is 7 times the number of dogs walked. Let the variable e represent earnings and the variable d represent the number of dogs walked.

\[ e = 7d \]

What It Means to You

You can use words, a table, a graph, or an equation to model the same mathematical relationship.

UNPACKING EXAMPLE 6.EE.9

The equation \( y = 4x \) represents the total cost \( y \) for \( x \) games of miniature golf. Make a table of values and a graph for this situation.

<table>
<thead>
<tr>
<th>Number of games, ( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total cost ($), ( y )</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
</tr>
</tbody>
</table>

Common Core Standards

<table>
<thead>
<tr>
<th>Lesson 12.1</th>
<th>Lesson 12.2</th>
<th>Lesson 12.3</th>
<th>Lesson 12.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMMON CORE</td>
<td>COMMON CORE</td>
<td>COMMON CORE</td>
<td>COMMON CORE</td>
</tr>
</tbody>
</table>

6.NS.6b Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane.

6.NS.6c Find and position integers and other rational numbers on a number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

6.NS.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and the independent variables using graphs and tables.
**Lesson 12.1 Graphing on the Coordinate Plane**

**Common Core Standards**
The student is expected to:

- **The Number System**
  - 6.NS.6c

Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. Also 6.NS.6, 6.NS.6b, 6.NS.8

**Mathematical Practices**
- **MP.2** Reasoning

**Engage**

**ESSENTIAL QUESTION**

*How do you locate and name points in the coordinate plane?* Sample answer: Points in the coordinate plane are located and named by positions to the left or right of the origin first, then above or below the origin. The order of the coordinates is important.

**Motivate the Lesson**

Ask: Have you ever tried to find a city or town by its location on a map grid? Maps are like a coordinate plane. Begin Example 1 to find out how to locate a point on a coordinate plane.

**Explore**

**Engage with the Whiteboard**

To introduce students to the four-quadrant coordinate plane, sketch a simple “treasure map” with a coordinate plane on the whiteboard. Mark a point for “Start” at the origin and a point for “Treasure” in Quadrant I. Ask students to draw a path to the treasure using the grid lines and then to describe the path in words, such as, “Walk east 3 steps. Then walk north 5 steps.” Repeat several times with new coordinates for the “Treasure.”

**Explain**

**EXAMPLE 1**

**Focus on Communication**

Point out to students that coordinates describe a location in relation to the origin, so it is important to always start at the origin when identifying the coordinates of a point.

**Questioning Strategies**

• Is point (2, 3) the same as point (3, 2)? Explain. No, they are not the same point. Point (2, 3) lies 2 units to the right of the origin and 3 units up, while point (3, 2) lies 3 units to the right and 2 units up.

**YOUR TURN**

**Avoid Common Errors**

Students may give incorrect coordinates for a point because they transposed the x- and y-coordinates. Remind students that the x-coordinate is the first number in an ordered pair.

**EXAMPLE 2**

**Connect Vocabulary**

Some students may have difficulty remembering what the x- and y-coordinates mean in an ordered pair. Encourage students to think of plotting points as physical movements, run and jump. The x-coordinate tells how far to run to the right or left, and the y-coordinate tells how far to jump up or down. So, when plotting points, students should always “run before they jump.”

**Questioning Strategies**

• Describe how graphing the point (0, −3) is similar to graphing the point (−3, 0). How is it different? Sample answer: They are similar because you start at the origin and move three units to graph each point. They are different because in (0, −3), you move down from the origin. In (−3, 0), you move left from the origin.
Lesson 12.1  Graphing on the Coordinate Plane

Essential Question: How do you locate and name points in the coordinate plane?

Naming Points in the Coordinate Plane

A coordinate plane is formed by two number lines that intersect at right angles. The point of intersection is 0 on each number line.

- The two number lines are called axes.
- The horizontal axis is called the x-axis.
- The vertical axis is called the y-axis.
- The point where the axes intersect is called the origin.
- The two axes divide the coordinate plane into four quadrants.

An ordered pair is a pair of numbers that gives the location of a point on a coordinate plane. The first number tells how far right (positive) or left (negative) the point is located from the origin. The second number tells how far up (positive) or down (negative) the point is located from the origin.

The numbers in an ordered pair are called coordinates. The first number is the x-coordinate and the second number is the y-coordinate.

Example 1

Identify the coordinates of each point. Name the quadrant where each point is located.

Point A is 1 unit left of the origin, and 5 units down. It has x-coordinate -1 and y-coordinate -5, written (-1, -5). It is located in Quadrant III.

Point B is 2 units right of the origin, and 3 units up. It has x-coordinate 2 and y-coordinate 3, written (2, 3). It is located in Quadrant I.

Reflect

1. If both coordinates of a point are negative, in which quadrant is the point located? ________

2. Describe the coordinates of all points in Quadrant I. Both coordinates are positive.

3. Communicate Mathematical Ideas Explain why (-3, 5) represents a different location than (3, 5).

The x-coordinate in (-3, 5) is -3 which will be to the left of the origin. The x-coordinate in (3, 5) is 3 which will be to the right of the origin.

Your Turn

Identify the coordinates of each point. Name the quadrant where each point is located.

4. G (4, -4); IV
5. E (-2, 4); II
6. F (3, 2); I
7. H (-1, -3); III

Graphing Points in the Coordinate Plane

Points that are located on the axes are not located in any quadrant. Points on the x-axis have a y-coordinate of 0, and points on the y-axis have an x-coordinate of 0.

Example 2

Graph and label each point on the coordinate plane.

A(-5, 2), B(3, 1.5), C(0, -3)

Point A is 5 units left and 2 units up from the origin.

Point B is 3 units right and 1.5 units up from the origin. Graph the point halfway between (3, 1) and (3, 2).

Point C is 3 units down from the origin. Graph the point on the y-axis.

Professional Development

Integrate Mathematical Practices MP.3

This lesson provides an opportunity to address this Mathematical Practice standard. It calls for students to create and use representations to organize, record, and communicate mathematical ideas. Students use coordinate planes to locate points. Then students solve a real-world problem on a coordinate plane in which the scale on each axis represents a real-world situation. In this way, students are able to connect a coordinate plane to the real world.

Math Background

The concept of the rectangular coordinate system is generally credited to French mathematician and philosopher René Descartes and, therefore, is sometimes referred to as the Cartesian plane. Every point on the plane can be located because all real numbers, not just integers, are used. The points represented by integer coordinates are sometimes called lattice points.
YOUR TURN
Avoid Common Errors
Students may graph the points incorrectly by using the x- and y-coordinates in the wrong order. Remind students to run before they jump.

EXAMPLE 3
Focus on Math Connections  Mathematical Practices
Point out to students that the coordinate plane also indicates directions. The x-axis points east (to the right) and west (to the left) and the y-axis points north (up) and south (down). For Example 3, have students add the correct north, south, east, and west labels to the axes of the coordinate plane.

Questioning Strategies  Mathematical Practices
• Describe the direction you would go from Gary’s house to Jen’s house. I would travel east to go from Gary’s house to Jen’s house.
• What would the coordinates of Gary’s house be if he lived 30 miles directly east of Jen? Explain. (35, 15); Jen lives at (5, 15), so 30 miles directly east would be (35, 15).

YOUR TURN
Focus on Math Connections  Mathematical Practices
Show students how to translate directions to movements by using the axes. Remind students that “20 miles south” is on the y-axis below the origin and that “20 miles west” is on the x-axis to the left of the origin. Both movements are in a negative direction, so the coordinates of Ted’s home are (−20, −20). Since Ned lives “50 miles directly north of Ted’s house” only the y-coordinate changes. Because north is a positive direction, −20 + 50 = 30. So, Ned’s home is located at (−20, 30).

Elaborate
Talk About It
Summarize the Lesson
Have students complete a graphic organizer that shows the number of each quadrant and the signs of the coordinates in each quadrant.

GUIDED PRACTICE
Engage with the Whiteboard
For Exercises 1–2, have students fill in the blanks by identifying the coordinates of each point on the whiteboard. Then have them name the quadrant where each point is located. Also ask students to describe how they would graph each point.

Avoid Common Errors
Exercises 3–4, 6–7  Students may graph the points incorrectly by using the x- and y-coordinates in the wrong order. Remind students to run before they jump.
**Reading Scales on Axes**

The scale of an axis is the number of units that each grid line represents. So far, the graphs in this lesson have a scale of 1 unit, but graphs frequently use other units.

**EXAMPLE 3**

The graph shows the location of a city. It also shows the location of Gary’s and Jen’s houses. The scale on each axis reports miles.

1. Use the scale to describe Gary’s location relative to the city.
   - Each grid square is 5 miles on a side.
   - Gary’s house is at (25, 15), which is 25 miles west and 15 miles north of the city.
2. Describe the location of Jen’s house relative to Gary’s house.
   - Jen’s house is located 6 grid squares to the right of Gary’s house.
   - Since each grid square is 5 miles on a side, her house is 6 \times 5 = 30 miles from Gary’s.

**YOUR TURN**

Graph and label each point on the coordinate plane.

6. \((-4, 2)\)
7. \((3, 2.5)\)
8. \((-4.5, -5)\)
9. \((4, -5)\)
10. \((1, -2, 5, 0)\)

**Guided Practice**

Identify the coordinates of each point in the coordinate plane. Name the quadrant where each point is located. (Example 1)

1. Point A is 5 units left of the origin and 1 unit up from the origin. Its coordinates are \((-5, 1)\). It is in quadrant II.
2. Point B is 2 units right of the origin and 3 units down from the origin. Its coordinates are \((2, -3)\). It is in quadrant IV.

Graph and label each point on the coordinate plane above. (Example 2)

3. Point C at \((-3.5, 3)\)
4. Point D at \((5, 0)\)

For 5–7, use the coordinate plane shown. (Example 3)

5. Describe the scale of the graph.
   - Each grid square is \(\frac{1}{2}\) unit on a side.
6. Plot point A at \((-\frac{1}{2}, 2)\).
7. Plot point B at \((\frac{1}{2}, -2)\).
8. Vocabulary Describe how an ordered pair represents a point on a coordinate plane. Include the terms x-coordinate, y-coordinate, and origin in your answer.
   - The first number, the x-coordinate, tells how many units to the right or left the point is located from the origin.
   - The second number, the y-coordinate, tells how many units up or down the point is located from the origin.

**ESSENTIAL QUESTION CHECK-IN**

9. Give the coordinates of one point in each of the four quadrants, one point on the x-axis, and one point on the y-axis.
   - Sample answers: Quadrant I: \((2, 5)\); Quadrant II: \((-3, 5)\); Quadrant III: \((-3, -3)\); Quadrant IV: \((5, -2)\).
   - x-axis: \((-3, 0)\); y-axis: \((0, 5)\).

**Mathematical Practices**

Math Talk

How are north, south, east, and west represented on the graph in Example 3?

**Cooperative Learning**

Have students work in three teams to play coordinate tic-tac-toe. Use a coordinate plane that is 5 units from the origin in all directions. One player on each team alternates calling out the coordinates of a point. Another player on each team locates the point and marks it on the coordinate plane. The first team to place three marks in an uninterrupted row horizontally, vertically, or diagonally wins the round.

**Additional Resources**

- Differentiated Instruction includes:
  - Reading Strategies
  - Success for English Learners ELL
  - Reteach
  - Challenge PRE-AP
Evaluate

**GUIDED AND INDEPENDENT PRACTICE**

**Concepts & Skills**

<table>
<thead>
<tr>
<th>Example</th>
<th>Practice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Naming Points in the Coordinate Plane</td>
<td>Exercises 1–2, 10–13</td>
</tr>
<tr>
<td>Graphing Points in the Coordinate Plane</td>
<td>Exercises 3–4, 10–13</td>
</tr>
<tr>
<td>Reading Scales on Axes</td>
<td>Exercises 5–7, 14–15</td>
</tr>
</tbody>
</table>

**Exercise** | **Depth of Knowledge (D.O.K.)** | **Mathematical Practices** |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2 Skills/Concepts</td>
<td>MP.4 Modeling</td>
</tr>
<tr>
<td>11</td>
<td>3 Strategic Thinking</td>
<td>MP.7 Using Structure</td>
</tr>
<tr>
<td>12–13</td>
<td>2 Skills/Concepts</td>
<td>MP.4 Modeling</td>
</tr>
<tr>
<td>14–15</td>
<td>2 Skills/Concepts</td>
<td>MP.2 Reasoning</td>
</tr>
<tr>
<td>16</td>
<td>3 Strategic Thinking</td>
<td>MP.8 Patterns</td>
</tr>
<tr>
<td>17</td>
<td>3 Strategic Thinking</td>
<td>MP.3 Logic</td>
</tr>
<tr>
<td>18–19</td>
<td>3 Strategic Thinking</td>
<td>MP.7 Using Structure</td>
</tr>
</tbody>
</table>

**Common Core** 6.NS.6b, 6.NS.6c, 6.NS.8

**Answers**

1. Henry (−3, 4), Library (2, −3)
2. Henry is 5 blocks west and 7 blocks north of the library.
3. (−5, −3)
4. (−1.5, −3)

**Additional Resources**

**Differentiated Instruction** includes:

- Leveled Practice Worksheets
12.1 Independent Practice

6.NS.6, 6.NS.6a, 6.NS.6c, 6.NS.8

For 10–13, use the coordinate plane shown. Each unit represents 1 kilometer.

10. Write the ordered pairs that represent the location of Sam and the theater.
   Sam: (4, 2); Theater: (−3, 5)

11. Describe Sam’s location relative to the theater.
   Sam is 3 km south and 7 km east of the theater.

12. Sam wants to meet his friend Beth at a restaurant before they go to the theater. The restaurant is 4 km south of the theater.
    Plot and label a point representing the restaurant. What are the coordinates of the point?
    (−3, −4)

13. Beth describes her current location: “I’m directly south of the theater, halfway to the restaurant!” Plot and label a point representing Beth’s location. What are the coordinates of the point?
    (−3, 0.5)

For 14–15, use the coordinate plane shown.

14. Find the coordinates of points T, U, and V.
    T (0.75, −1.0); U (0, 1.25); V (0.75, 1.25)

15. Points T, U, and V are the vertices of a rectangle. Point W is the fourth vertex. Plot point W and give its coordinates.
    W (−0.75, −1.0)

16. Explain the Error. Janine tells her friend that ordered pairs that have an x-coordinate of 0 lie on the x-axis. She uses the origin as an example. Describe Janine’s error. Use a counterexample to explain why Janine’s statement is false.
   Janine is describing points that lie on the y-axis. Ordered pairs that lie on the x-axis have a y-coordinate of 0. The origin lies on the x- and y-axis. Any other point with an x-coordinate of 0, such as (0, 3), lies on the y-axis.

EXTEND THE MATH

Activity Plot the points for each set of ordered pairs below. Then connect the points in the order shown to reveal a figure. Name the figure and find its area.

Set 1: (2, 5), (2, −1), (−3, −1), (−3, 5)
Set 2: (−4, −3), (6, −3), (6, 4)
Set 3: (1, 3), (−4, 3), (−4, −2), (1, −2)

Write the coordinates for another set of points that form a figure. Find its area.

Then challenge a classmate to draw the figure and find its area.

Set 1: rectangle, $A = 30$ square units
Set 2: triangle, $A = 35$ square units
Set 3: square, $A = 25$ square units

17. Critical Thinking Choose scales for the coordinate plane shown so that you can graph the points J(2, 40), K(3, 10), L(3, −40), M(−4, 50), and N(−5, −50). Explain why you chose the scale for each axis.

Sample answer: On the x-axis I used a scale of 1 unit for each grid square. On the y-axis I used a scale of 10 units for each grid square.

The x-coordinates ranged from −5 to 3, and the y-coordinates ranged from −50 to 50.

18. Communicate Mathematical Ideas Edgar wants to plot the ordered pair (1.8, −1.2) on a coordinate plane. On each axis, one grid square equals 0.1. Starting at the origin, how can Edgar find (1.8, −1.2)?
   Count 18 grid squares to the right (in a positive direction) along the x-axis. Then count 12 grid squares down (in a negative direction).

19. Represent Real-World Problems Zach graphs some ordered pairs in the coordinate plane. The x-values of the ordered pairs represent the number of hours since noon, and the y-values represent the temperature at that time.
   a. In which quadrants could Zach graph points? Explain your thinking.
      Quadrants I and IV are always positive, but temperatures can be positive or negative. In quadrants I and IV, the x-coordinate is always positive, but the y-coordinate can be positive or negative.
   b. In what part of the world and at what time of year might Zach collect data so that the points he plots are in Quadrant IV?
      Sample answer: in a region with a cold climate during the winter
Engage

ESSENTIAL QUESTION

How can you identify independent and dependent quantities from tables and graphs?

Sample answer: The dependent variable is the quantity that depends on the other variable. On a graph, the independent variable is shown on the horizontal axis and the dependent variable is shown on the vertical axis.

Motivate the Lesson

Ask: Suppose a person gets paid by the hour. What is the relationship between the amount of time the person works and the amount of money that person earns? Begin the Explore Activity to find out what independent and dependent quantities are and how to recognize them.

Explore

EXPLORE ACTIVITY 1

Connect to Vocabulary

Have students describe the meaning of the following phrases: Sample answers are given.

• independently wealthy doesn't need to work for money
• Independence Day day of freedom
• working independently doesn't need help to do a job
• dependent child needs a parent
• insulin-dependent needs insulin daily
• dependent clause cannot stand alone in a sentence

Then ask students to define independent and dependent variables. An independent variable stands alone and isn’t changed by the other variables. A dependent variable depends on or is changed by another variable.

Explain

EXPLORE ACTIVITY 2

Connect Multiple Representations

Have students complete this table, which represents the situation about the art teacher and clay, to reinforce that both a table and a graph can represent this relationship.

<table>
<thead>
<tr>
<th>Clay bought by teacher (lb)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay available for classes (lb)</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
</tr>
</tbody>
</table>

Engage with the Whiteboard

Ask a student volunteer to locate the point on the graph that shows the 50 pounds of clay that is available for the art class. Have the volunteer draw a line from the y-axis to the point and a line from the point to the x-axis. Have students repeat this process for several more values.
**ESSENTIAL QUESTION**
How can you identify independent and dependent quantities from tables and graphs?

**EXPLORE ACTIVITY 1**

**Identifying Independent and Dependent Quantities from a Table**

Many real-world situations involve two variable quantities in which one quantity depends on the other. The quantity that depends on the other quantity is called the dependent variable, and the quantity it depends on is called the independent variable.

A freight train moves at a constant speed. The distance $y$ in miles that the train has traveled after $x$ hours is shown in the table.

<table>
<thead>
<tr>
<th>Time $x$ (h)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance $y$ (mi)</td>
<td>0</td>
<td>50</td>
<td>100</td>
<td>150</td>
</tr>
</tbody>
</table>

**A** What are the two quantities in this situation?  
*time and distance*

Which of these quantities depends on the other?  
*Distance depends on time.*

What is the independent variable?  
*time, $x$*

What is the dependent variable?  
*distance, $y$*

**B** How far does the train travel each hour?  
*50 miles*

The relationship between the distance traveled by the train and the time in hours can be represented by an equation in two variables.

$$
\begin{array}{c|c|c|c|}
\text{Distance traveled (miles)} & \text{Distance traveled per hour} & \text{Time (hours)} \\
\hline
\downarrow & \downarrow & \downarrow \\
y & 50 & x \\
\end{array}
$$

**EXPLORE ACTIVITY 2**

**Identifying Independent and Dependent Variables from a Graph**

**A** If the teacher buys 10 more pounds of clay, how many pounds will be available for the art class?  
*30 lb*

If the art class has a total of 50 pounds of clay available, how many pounds of clay did the teacher buy?  
*30 lb; find the point on the graph with a $y$-coordinate of 50. Then find the $x$-coordinate of this point, which is 30.*

**Reflect**

1. **Analyze Relationships** Describe how the value of the independent variable is related to the value of the dependent variable. Is the relationship additive or multiplicative?  
*The value of $y$ is always 50 times the value of $x$; multiplicative.*

2. **What are the units of the independent variable and of the dependent variable?**  
*Independent variable: hours; dependent variable: miles.*

3. **A rate is used in the equation. What is the rate?**  
*50 miles per hour*

**PROFESSIONAL DEVELOPMENT**

**Integrate Mathematical Practices MP.4**

This lesson provides an opportunity to address this Mathematical Practice standard. It calls for students to communicate mathematical ideas using multiple representations as appropriate. Students use tables, graphs, equations, and language to describe and model relationships between independent and dependent variables. In this way, students use multiple representations to model real-world situations involving independent and dependent variables.

**Math Background**

Although the term *function* is not mentioned in this lesson, the tables in the lesson represent functions. A function is a rule that relates two quantities so that each input value corresponds to exactly one output value. When $y$ is a function of $x$, $x$ is called the independent variable and $y$ is called the dependent variable. Whenever a value is assigned to $x$, a value is automatically assigned to $y$ by an applicable rule or correspondence.
EXPLORE ACTIVITY 2 (CONTINUED)

Questioning Strategies Mathematical Practices

- Why does the graph show only Quadrant I? Negative amounts do not make sense in this situation, so the values and the graph are limited to positive x- and y-values.
- Why does the graph start at (0, 20)? The art teacher had 20 pounds of clay to start with.
- As the x-value is increasing, what is happening to the y-value? The y-value is also increasing.

EXAMPLE 1

Engage with the Whiteboard

Have a volunteer sketch a graph of the relationship shown in the table in A. Have a second volunteer make a table of the relationship shown in the graph in B. This will help students to see that both a table and a graph can represent the same relationship.

Connect Multiple Representations Mathematical Practices

Point out to students that each of these situations can be represented by a verbal description, a table, a graph, or an equation.

Questioning Strategies

- How can the relationship in A be represented by an equation? The table begins with a y-value of 10, so the y-value always will be 10 units greater than the x-value. Then, as x increases by 1, y also increases by 1, resulting in the equation $y = x + 10$.
- How would you describe the relationship in A? Explain. The relationship is an additive relationship because the value of $y$ is always 10 units greater than the value of $x$.
- How can the relationship in B be represented by an equation? The graph begins at the origin, so both variables begin at 0. Then, as x increases by 1, y increases by 12, resulting in the equation $y = 12x$.
- How could you check that the equation is correct for either A or B? Pick a point from either the table or the graph and substitute it into the equation. The result should be a true equation.

Focus on Reasoning Mathematical Practices

Ask students to identify the independent and dependent quantities in the following situations.

- A veterinarian must weigh an animal before determining the amount of medication it needs. independent quantity: weight of animal, dependent quantity: amount of medication
- A company charges $10 per hour to rent a jackhammer. independent quantity: time, dependent quantity: cost

ADDITIONAL EXAMPLE 1

A The table below shows a relationship between two variables, x and y. Describe a possible situation the table could represent. Describe the independent and dependent variables in this situation.

<table>
<thead>
<tr>
<th>Independent variable, x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable, y</td>
<td>8</td>
<td>16</td>
<td>24</td>
<td>32</td>
</tr>
</tbody>
</table>

Sample answer: The table could represent the amount a person earns at a rate of $8 per hour. The independent variable, x, is the number of hours the person works. The dependent variable, y, is the total earnings.

B The graph below shows a relationship between two variables, x and y. Describe a possible situation the graph could represent. Describe the independent and dependent variables.

Sample answer: The graph could represent the progress of a rock climber, starting at a 2-foot height and continuing at a pace of 1 foot every second. The independent variable is the number of seconds, and the dependent variable is the total number of feet climbed after x seconds.

Interactive Whiteboard

Interactive example available online
**Curriculum Integration**

**Music:** The notes you hear played by a musical instrument are an example of a dependent relationship. For example, a clarinet’s pitch at a particular moment depends on the number of holes covered by the musician. A harp’s pitch depends on the length of the string being plucked.

---

**Cooperative Learning**

One way to remember which is the independent variable and which is the dependent variable is to use the names of the two variables in a sentence that makes sense. For example:

*Dollars Earned* depends on *Hours Worked*, but *Hours Worked* does not depend on *Dollars Earned*. So, *Dollars Earned* must be the dependent variable and *Hours Worked* must be the independent variable.

---

**Additional Resources**

**Differentiated Instruction** includes:

- Reading Strategies
- Success for English Learners
- Reteach
- Challenge

---

**Reflect**

4. In this situation, the same units are used for the independent and dependent variables. How is this different from the situation involving the train in the first Explore? The other situation involves two different units (miles and hours).

5. **Analyze Relationships** Tell whether the relationship between the independent variable and the dependent variable is a multiplicative or an additive relationship.

6. What are the units of the independent variable, and what are the units of the dependent variable?

   - Independent variable: **pounds**
   - Dependent variable: **pounds**

---

**EXAMPLE 1**

**The table shows a relationship between two variables, x and y. Describe a possible situation the table could represent. Describe the independent and dependent variables in the situation.**

<table>
<thead>
<tr>
<th>Independent variable, x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable, y</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
</tr>
</tbody>
</table>

As x increases by 1, y increases by 1. The relationship is additive.

The table could represent Jina’s savings if she starts with $10 and adds $1 to her savings every day.

The independent variable, x, is the number of days she has been adding money to her savings.

The dependent variable, y, is her savings after x days.

The graph shows a relationship between two variables. Describe a possible situation that the graph could represent. Describe the independent and dependent variables.

As x increases by 1, y increases by 12. The relationship is multiplicative. The value of y is always 12 times the value of x.

The graph could represent the number of eggs in cartons that each hold 12 eggs.

The independent variable, x, is the number of cartons.

The dependent variable, y, is the total number of eggs.

---

**Reflect**

7. What are other possible situations that the table and graph in the Examples could represent?

Sample answer table: Paul has 10 DVDs and buys more.

Independent: number of DVDs he buys; dependent: number he has after he buys x DVDs.

Graph: 12 photos fit on each page of a yearbook. Independent: number of pages; dependent: total number of photos on x pages.
YOUR TURN

Avoid Common Errors
If students have difficulty distinguishing between independent and dependent variables, remind them that the independent variable causes a change in the dependent variable, while the dependent variable could not cause a change in the independent variable.

Elaborate

Talk About It
Summarize the Lesson

Ask: How do you know which is the dependent variable and which is the independent variable in a table or graph? In a table, the independent variable usually is represented by the variable x. The dependent variable usually is represented by the variable y. On a graph, the independent variable usually is shown on the horizontal axis and the dependent variable on the vertical axis.

GUIDED PRACTICE

Engage with the Whiteboard

For Exercises 1–2, have a student sketch the graph to represent the table on the whiteboard. Have the student explain how to know which quantity should be represented by the x-axis and which by the y-axis.

For Exercise 3, have a student volunteer label the axes on the graph to represent the real-world situation suggested by the student.

Avoid Common Errors

Exercise 3 If students have difficulty determining whether a relationship is additive or multiplicative, remind them that in a multiplicative relationship the graph will pass through the origin, but in an additive relationship the graph will not pass through the origin.

Exercise 4 Remind students that if the independent variable is on the horizontal axis of a graph, the dependent variable is on the vertical axis of the graph.
1. A boat rental shop rents paddleboats for a fee plus an additional cost per hour. The cost of renting for different numbers of hours is shown in the table.

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($)</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
</tr>
</tbody>
</table>

What is the independent variable, and what is the dependent variable? How do you know? (Explore Activity 1)

Time is the independent variable and cost is the dependent variable, because cost depends on the number of hours rented.

2. A car travels at a constant rate of 60 miles per hour. (Explore Activity 1)

<table>
<thead>
<tr>
<th>Time x (h)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance y (mi)</td>
<td>0</td>
<td>60</td>
<td>120</td>
<td>180</td>
</tr>
</tbody>
</table>

a. Complete the table.

b. What is the independent variable, and what is the dependent?

Time is the independent variable and distance is the dependent variable.

c. Describe how the value of the independent variable is related to the value of the dependent variable.

The value of $y$ is always 60 times the value of $x$.

Use the graph to answer the questions.

3. Describe in words how the value of the independent variable is related to the value of the dependent variable. (Explore Activity 2)

The value of the dependent variable is 5 times the value of the independent variable.

4. Describe a real-world situation that the graph could represent. (Example 1)

Sample answer: The graph could represent the total cost $y$ of buying $x$ carnival tickets for $5 each.

<table>
<thead>
<tr>
<th>Time</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
</tr>
</tbody>
</table>

Sample answer: Bridget’s grandmother gave her a collection of 15 perfume bottles. Bridget adds one bottle per week to the collection. The independent variable is the number of weeks. The dependent variable is the number of perfume bottles in her collection. The value of $y$ is always 15 units greater than the value of $x$.

Sample answer: Colin created a website to sell T-shirts that are printed with funny slogans. He makes $16 per T-shirt. The independent variable is the number of T-shirts he sells, and the dependent variable is his profit in dollars. As the independent variable increases by 1, the dependent variable increases by 16.

Sample answer: Tickets to the school musical cost $3 each. The independent variable is the number of tickets purchased, and the dependent variable is the total cost. The value of $y$ is always 3 times the value of $x$.

Sample answer: The dependent variable is the quantity that depends on the other variable. On a graph, the independent variable is usually shown on the horizontal axis and the dependent variable on the vertical axis.
Evaluate

GUIDED AND INDEPENDENT PRACTICE

<table>
<thead>
<tr>
<th>Concepts &amp; Skills</th>
<th>Practice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explore Activity 1</td>
<td>Exercises 1–2, 7</td>
</tr>
<tr>
<td>Identifying Independent and Dependent Quantities from a Table</td>
<td></td>
</tr>
<tr>
<td>Explore Activity 2</td>
<td>Exercises 3, 6, 8</td>
</tr>
<tr>
<td>Identifying Independent and Dependent Variables from a Graph</td>
<td></td>
</tr>
<tr>
<td>Example 1</td>
<td>Exercises 4, 6</td>
</tr>
<tr>
<td>Describing Relationships Between Independent and Dependent Variables</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Depth of Knowledge (D.O.K.)</th>
<th>Mathematical Practices</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>2 Skills/Concepts</td>
<td>MP.4 Modeling</td>
</tr>
<tr>
<td>7</td>
<td>3 Strategic Thinking</td>
<td>MP.7 Using Structure</td>
</tr>
<tr>
<td>8–9</td>
<td>3 Strategic Thinking</td>
<td>MP.3 Logic</td>
</tr>
<tr>
<td>10</td>
<td>3 Strategic Thinking</td>
<td>MP.8 Patterns</td>
</tr>
</tbody>
</table>

Additional Resources
Differentiated Instruction includes:
- Leveled Practice Worksheets

Answers
1. Independent variable, \( x \), is the number of tickets bought; dependent variable, \( y \), is the total cost.
2. It is an additive relationship.
3. The total cost is the number of tickets bought plus $5 for postage for the tickets.

4. Sample answer: Parking at the airport costs $7 per day. Independent variable, \( x \), is the number of days a vehicle is parked; dependent variable, \( y \), is the total cost for parking.
6. The graph shows the relationship between the hours a soccer team practiced after the season started and their total practice time for the year.
   a. How many hours did the soccer team practice before the season began?
   6 hours
   b. What are the two quantities in this situation?
      hours practiced during the season and total practice time for year
   c. What are the dependent and independent variables?
      independent: hours practiced during season; dependent: total practice time for year
   d. Is the relationship between the variables additive or multiplicative? Explain.
      Additive; the total practice increases by 1 hour as the practice time during the season increases by 1 hour.
   e. Analyze Relationships Describe the relationship between the quantities in words.
      Total practice time for the year is 6 hours more than practice time during the season.

7. Multistep Teresa is buying glitter markers to put in gift bags. The table shows the relationship between the number of gift bags and the number of glitter markers she needs to buy.
   a. What is the dependent variable? number of markers
   b. What is the independent variable? number of gift bags
   c. Is the relationship additive or multiplicative? Explain.
      The relationship is multiplicative because y increases by a factor of 5 as x increases by 1.
   d. Describe the relationship between the quantities in words.
      The number of glitter markers is 5 times the number of gift bags.

8. Ty borrowed $500 from his parents. The graph shows how much he owes them each month if he pays back a certain amount each month.
   a. Describe the relationship between the number of months and the amount Ty owes. Identify an independent and dependent variable and explain your thinking.
      Ty starts out owing $500 and every month the amount he owes decreases by $50; independent variable: number of months; dependent variable: amount he owes; the amount he owes depends on the number of months he has been paying.
   b. How long will it take Ty to pay back his parents?
      10 months

9. Error Analysis A discount store has a special: 8 cans of juice for a dollar. A shopper decides that since the number of cans purchased is 8 times the number of dollars spent, the cost is the independent variable and the number of cans is the dependent variable. Do you agree? Explain.
   Sample answer: I disagree because the amount a shopper pays depends on the number of cans purchased. So, the number of cans is the independent variable, and cost is the dependent variable.

10. Analyze Relationships Provide an example of a real-world relationship where there is no clear independent or dependent variable. Explain.
    Sample answer: Andrea is 4 years older than Lisa. You could say that Andrea’s age depends on Lisa’s because you can add 4 to Lisa’s age. You can also say that Lisa’s age depends on Andrea’s age because you can subtract 4 from Andrea’s age.

**EXTEND THE MATH**

Introduce students to independent and dependent variables in situations that involve decimals or fractions. For example:

Gina is charged $0.15 for each text message that she sends.

1. What is the independent variable? a
   a. number of texts sent
   b. charge per text
   c. total amount charged for texting

2. What is the dependent variable? c
   a. number of texts sent
   b. charge per text
   c. total amount charged for texting

3. Write an equation that expresses the situation.
   Let b be the amount of Gina’s bill.
   Let s be the number of texts sent.
   \[ b = 0.15s \]
LESSON 12.3 Writing Equations from Tables

Common Core Standards
The student is expected to:

**Expressions and Equations—6.EE.9**
Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

Mathematical Practices

**MP.1 Problem Solving**

---

**Engage**

**ESSENTIAL QUESTION**

*How can you use an equation to show a relationship between two variables?* Use a table to find the relationship between the two variables. Use that relationship to write an equation.

**Motivate the Lesson**

Ask students to imagine a hot dog stand that charges $3 per hot dog. How much would 4 hot dogs cost? 30 hot dogs? Begin the Explore Activity to find out how to write an equation that will help you predict the total cost for any number of hot dogs.

---

**Explore**

**EXPLORE ACTIVITY**

**Focus on Patterns**

Point out to students that to write an equation from the data in the table, they need to look for a pattern in the data. First, they should look for changes in both the input values and the output values. Then they need to see how the changes are related. For example:

- \( 8 = 8 \cdot 1 \)
- \( 16 = 8 \cdot 2 \)
- \( 24 = 8 \cdot 3 \)

So, the pattern is \( y = 8 \cdot x \), where \( x \) is the number of dogs walked and \( y \) is the amount of money earned.

---

**Explain**

**EXAMPLE 1**

**Focus on Reasoning**

In A, point out to students that the \( y \)-value is always less than the \( x \)-value. Therefore, the operation in the equation must be subtraction, division, or multiplication by a factor that is less than 1. In B, point out to students that the \( y \)-value is always more than the \( x \)-value. Therefore, the operation in the equation must be addition or multiplication by a factor that is greater than 1.

**Questioning Strategies**

- For A, how can you write an equation that expresses \( x \) in terms of \( y \)? I can compare the \( x \)- and \( y \)-values and find that each \( x \)-value is twice the corresponding \( y \)-value, which gives me the equation \( x = 2y \).

**YOUR TURN**

**Engage with the Whiteboard**

For Exercises 2–5, have students write a pattern on the whiteboard for each table. Then have them use the pattern to write an equation to represent each table. Ask students to explain their reasoning.
LESSON 12.3 Writing Equations from Tables

ESSENTIAL QUESTION
How can you use an equation to show a relationship between two variables?

EXPLORE ACTIVITY
Writing an Equation to Represent a Real-World Relationship

Many real-world situations involve two variable quantities in which one quantity depends on the other. This type of relationship can be represented by a table. You can also use an equation to model the relationship.

The table shows how much Amanda earns for walking 1, 2, or 3 dogs. Use the table to determine how much Amanda earns per dog. Then write an equation that models the relationship between number of dogs walked and earnings. Use your equation to complete the table.

<table>
<thead>
<tr>
<th>Dogs walked</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earnings</td>
<td>$8</td>
<td>$16</td>
<td>$24</td>
<td>$40</td>
<td>$80</td>
<td>$160</td>
</tr>
</tbody>
</table>

A. For each column, compare the number of dogs walked and earnings. What is the pattern?
   Each earnings amount is 8 times the corresponding number of dogs walked.

B. Based on the pattern, Amanda earns $8 for each dog she walks.

C. Write an equation that relates the number of dogs Amanda walks to the amount she earns. Let e represent earnings and d represent dogs.
   \[ e = 8 \cdot d \]

D. Use your equation to complete the table for 5, 10, and 20 walked dogs.

E. Amanda’s earnings depend on the number of dogs walked.

Reflect

1. What if? If Amanda changed the amount earned per dog to $11, what equation could you write to model the relationship between number of dogs walked and earnings?
   \[ e = 11 \cdot d \]

Writing an Equation Based on a Table

The relationship between two variables where one variable depends on the other can be represented in a table or by an equation. An equation expresses the dependent variable in terms of the independent variable.

When there is no real-world situation to consider, we usually say x is the independent variable and y is the dependent variable. The value of y depends on the value of x.

EXAMPLE 1

Write an equation that expresses y in terms of x.

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0.5</td>
<td>1</td>
<td>1.5</td>
<td>2</td>
<td>2.5</td>
</tr>
</tbody>
</table>

STEP 1
Compare the x- and y-values to find a pattern.
   Each y-value is \( \frac{1}{2} \) or 0.5 times, the corresponding x-value.

STEP 2
Use the pattern to write an equation expressing y in terms of x.
   \[ y = 0.5x \]

For each table, write an equation that expresses y in terms of x.

2. \[ y = x + 3 \]

3. \[ y = x - 2 \]

4. \[ y = x + 5 \]

5. \[ y = 2x \]

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices

This lesson provides an opportunity to address Mathematical Practices MP.4, which calls for students to “model with mathematics.” Students use verbal descriptions to make tables and draw graphs that represent real-life situations. They also represent information from graphs by using tables and equations; and represent equations by using tables and graphs.

Math Background

A rule that relates the x- and y-values in a table also can be called a relation. A relation describes a function if, for each x-value (input), there is only one y-value (output).

There are several different ways to describe the variables of a function:

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Dependent Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>x-value</td>
<td>y-value</td>
</tr>
<tr>
<td>Domain</td>
<td>Range</td>
</tr>
<tr>
<td>Input</td>
<td>Output</td>
</tr>
<tr>
<td>x</td>
<td>f(x)</td>
</tr>
</tbody>
</table>
ADDITIONAL EXAMPLE 2
Meredith is playing a video game. She earns the same number of points for each alien she captures. She earned 750 points for capturing 5 aliens and 1,350 points for capturing 9 aliens. Write an equation to represent the relationship. Then solve the equation to find how many points Meredith will earn if she captures 27 aliens.

\[ p = 150a, \] where \( p \) represents the number of points and \( a \) represents the number of aliens captured; 4,050 points

EXAMPLE 2

Focus on Reasoning Mathematical Practices
Point out to students that sometimes a problem may provide clues and facts that you must use to find a solution. Encourage students to begin by identifying the important information. They can underline or circle the information in the problem statement.

Engage with the Whiteboard
Have students extend the table on the whiteboard, continuing with sale prices of $400 through $1,200 in increments of $100. Then have students find the donation amount for these sale prices. After they have completed the table, ask students to identify any patterns in the table.

Questioning Strategies
• What is the independent variable in this situation? the dependent variable? The independent variable is the sale price of a painting. The dependent variable is the amount donated to charity.

• How else could you solve this problem? Use proportional reasoning. For instance, consider \( \frac{50}{200} = \frac{d}{1,200} \). Because \( 1,200 \div 200 = 6 \), multiply 50 by 6: \( d = 300 \).

YOUR TURN
Avoid Common Errors
If students have difficulty identifying the independent and dependent variables, remind them to begin by using the given information to make a table and then look for a pattern.

Elaborate

Talk About It
Summarize the Lesson
Ask: How can you use a table to write an equation that represents the relationship in the table? In the table, find the relationship between the independent and dependent variables. Then write the equation that represents the relationship.

GUIDED PRACTICE

Engage with the Whiteboard
For Exercises 1–2, have students write a pattern on the whiteboard for each table. Then have students use the pattern to write an equation to represent each table. Ask students to explain their reasoning.

For Exercise 5, have students fill in the missing information in the table on the whiteboard. Then have them identify the pattern and write an equation.

Avoid Common Errors
Exercises 1–4 Some students may write an equation that expresses \( x \) in terms of \( y \) instead of \( y \) in terms of \( x \). Remind them that the form of the equation should be \( y = kx \) or \( y = x + b \).

Exercise 5 If students have difficulty identifying the independent and dependent variables, remind them to begin by using the given information to make a table and then look for a pattern.
Using Tables and Equations to Solve Problems

You can use tables and equations to solve real-world problems.

**EXAMPLE 2**

A certain percent of the sale price of paintings at a gallery will be donated to charity. The donation will be $50 if a painting sells for $200. The donation will be $75 if a painting sells for $300. Find the amount of the donation if a painting sells for $1,200.

**Analyze Information**

You know the donation amount when the sale price of a painting is $200 and $300. You need to find the donation amount if a painting sells for $1,200.

**Formulate a Plan**

You can make a table to help you determine the relationship between sale price and donation amount. Then you can write an equation that models the relationship. Use the equation to find the unknown donation amount.

**Solve**

Make a table.

<table>
<thead>
<tr>
<th>Sale price ($)</th>
<th>200</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Donation amount ($)</td>
<td>50</td>
<td>75</td>
</tr>
</tbody>
</table>

You can make a table to help you determine the relationship between sale price and donation amount. Then you can write an equation that models the relationship. Use the equation to find the unknown donation amount.

<table>
<thead>
<tr>
<th>Sale price ($)</th>
<th>200</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Donation amount ($)</td>
<td>50</td>
<td>75</td>
</tr>
</tbody>
</table>

Write an equation. Let $p$ represent the sale price of the painting. Let $d$ represent the donation amount to charity.

The donation amount is equal to 25% of the sale price.

$$d = 0.25 \cdot p$$

Find the donation amount when the sale price is $1,200.

$$d = 0.25 \cdot 1,200$$

$$d = 300$$

Simplify to find the donation amount.

When the sale price is $1,200, the donation to charity is $300.

**Justify and Evaluate**

Substitute values from the table for $p$ and $d$ to check that they are solutions of the equation $d = 0.25 \cdot p$. Then check your answer of $300 by substituting for $d$ and solving for $p$.

$$d = 0.25 \cdot p$$

$$d = 0.25 \cdot 200$$

$$d = 50$$

$$d = 75$$

$$p = 1,200$$

One way to determine the relationship between sale price and donation amount is to find the percent.

$p$ is the independent variable; its value does not depend on any other value. $d$ is the dependent variable; its value depends on the price of the painting.

**Guided Practice**

Write an equation to express $y$ in terms of $x$. (Explore Activity, Example 1)

1. $\begin{array}{c|c|c|c}
   x & 10 & 20 & 30 \\
   \hline
   y & 6 & 16 & 26 \\
\end{array}$

   $y = x - 4$

2. $\begin{array}{c|c|c|c}
   x & 0 & 1 & 2 \\
   \hline
   y & 0 & 4 & 8 \\
\end{array}$

   $y = 4x$

3. $\begin{array}{c|c|c|c}
   x & 4 & 6 & 8 \\
   \hline
   y & 7 & 9 & 11 \\
\end{array}$

   $y = x + 3$

4. $\begin{array}{c|c|c|c}
   x & 12 & 24 & 36 \\
   \hline
   y & 2 & 4 & 6 \\
\end{array}$

   $y = \frac{x}{2}$

5. Jameson downloaded one digital song for $1.35, two digital songs for $2.70, and 5 digital songs for $6.75. Write and solve an equation to find the cost to download 25 digital songs. (Example 2)

   **Songs downloaded**
<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
</table>
   **Total cost ($)** | 1.35 | 2.70 | 6.75 | 13.50 |
   Number of songs = $n$; Cost = $c = 1.35n$
   The total cost of 25 songs is $33.75$

**ESSENTIAL QUESTION CHECK-IN**

6. Explain how to use a table to write an equation that represents the relationship in the table.

   Compare the $x$- and $y$-values to find a pattern. Use the pattern to write an equation expressing $y$ in terms of $x$.

**DIFFERENTIATE INSTRUCTION**

**Curriculum Integration**

Discuss the relationship between Celsius temperature and Kelvin temperature. Show students the following table and ask them to write an equation to convert from degrees Celsius to degrees Kelvin.

<table>
<thead>
<tr>
<th>Celsius (°C)</th>
<th>Kelvin (°K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-100</td>
<td>173</td>
</tr>
<tr>
<td>-50</td>
<td>223</td>
</tr>
<tr>
<td>0</td>
<td>273</td>
</tr>
<tr>
<td>50</td>
<td>323</td>
</tr>
<tr>
<td>100</td>
<td>373</td>
</tr>
</tbody>
</table>

$$K = C + 273$$

**Cognitive Strategies**

Some students may find it helpful to include a “Process” column in a table to help them identify patterns. Have students complete the table below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>Process</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-2 = -3 + 1</td>
<td>-2</td>
</tr>
<tr>
<td>-2</td>
<td>-1 = -2 + 1</td>
<td>-1</td>
</tr>
<tr>
<td>-1</td>
<td>0 = -1 + 1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1 = 0 + 1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2 = 1 + 1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3 = 2 + 1</td>
<td>3</td>
</tr>
</tbody>
</table>

Each value of $y$ is one more than the value of $x$.

**Additional Resources**

**Differentiated Instruction** includes:

- Reading Strategies
- Success for English Learners
- Reteach
- Challenge

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12.3 LESSON QUIZ

Write an equation that expresses $y$ in terms of $x$.

1.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
</tr>
</tbody>
</table>

2.

<table>
<thead>
<tr>
<th>$x$</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>7</td>
<td>17</td>
<td>27</td>
<td>37</td>
<td>47</td>
</tr>
</tbody>
</table>

3. Jaime bought 2 puzzles for $5.00 and 3 puzzles for $7.50. Write and solve an equation to find the cost of 15 puzzles.

4. A balloon rises to 100 feet in 4 minutes and 125 feet in 5 minutes. Write and solve an equation to find the distance the balloon rises in 8 minutes.

Lesson Quiz available online

Answers
1. $y = 5x$
2. $y = x - 3$
3. $c = 2.50 \cdot p; \$37.50$
4. $d = 25m; 200$ feet

Evaluate

GUIDED AND INDEPENDENT PRACTICE

Concepts & Skills | Practice
---|---
Explore Activity Writing an Equation to Represent a Real-World Relationship | Exercises 1–4, 8, 11
Example 1 Writing an Equation Based on a Table | Exercises 1–4, 9–10
Example 2 Using Tables and Equations to Solve Problems | Exercises 5, 11

Exercise | Depth of Knowledge (D.O.K.) | Mathematical Practices
---|---|---
7 | 3 Strategic Thinking | MP.7 Using Structure
8 | 2 Skills/Concepts | MP.4 Modeling
9–10 | 3 Strategic Thinking | MP.8 Patterns
11 | 3 Strategic Thinking | MP.4 Modeling
12 | 3 Strategic Thinking | MP.3 Logic
13–14 | 3 Strategic Thinking | MP.7 Using Structure
15 | 3 Strategic Thinking | MP.6 Precision

Additional Resources
Differentiated Instruction includes:
- Leveled Practice worksheets
7. **Vocabulary** What does it mean for an equation to express \( y \) in terms of \( x \)?

The variable \( y \) is on one side of the equation. The expression on the other side of the equation shows the relationship between \( x \) and \( y \).

8. The length of a rectangle is 2 inches more than twice its width.

Write an equation relating the length \( l \) of the rectangle to its width \( w \).

\[ l = 2w + 2 \]

9. **Look for a Pattern** Compare the \( y \)-values in the table to the corresponding \( x \)-values. What pattern do you see? How is this pattern used to write an equation that relates the \( x \)- and \( y \)-values?

<table>
<thead>
<tr>
<th>( x )</th>
<th>20</th>
<th>24</th>
<th>28</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

The \( y \)-value is \( \frac{1}{2} \) of the \( x \)-value. Write an equation that relates \( y \) to \( \frac{1}{2} \) of \( x \).

10. **Explain the Error** A student modeled the relationship in the table with the equation \( y = 4x \). Explain the student's error. Write an equation that correctly models the relationship.

<table>
<thead>
<tr>
<th>( x )</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>8</td>
<td>16</td>
<td>24</td>
<td>32</td>
</tr>
</tbody>
</table>

The student switched the variables; \( y = 4x \)

11. **Multistep** Marvin earns $8.25 per hour at his summer job. He wants to buy a video game system that costs $206.25.

   a. Write an equation to model the relationship between number of hours worked \( h \) and amount earned \( e \).

   \[ e = 8.25h \]

   b. Solve your equation to find the number of hours Marvin needs to work in order to afford the video game system.

\[ 206.25 = 8.25h; 25 = h; 25 \text{ hours} \]

12. **Communicate Mathematical Ideas** For every hour that Noah studies, his test score goes up 3 points. Explain which is the independent variable and which is the dependent variable. Write an equation modeling the relationship between hours studied \( h \) and the increase in Noah's test score \( s \).

   Independent: hours studied; its value does not depend on another variable; dependent: test score; its value depends on the number of hours that Noah studies; \( s = 3h \)

13. **Make a Conjecture** Compare the \( y \)-values in the table to the corresponding \( x \)-values. Determine whether there is an additive relationship or a multiplicative relationship between \( x \) and \( y \). If possible, write an equation modeling the relationship.

   Not possible; there is no consistent pattern between the \( y \)-values and corresponding \( x \)-values.

14. **Represent Real-World Problems** Describe a real-world situation in which there is an additive or multiplicative relationship between two quantities. Make a table that includes at least three pairs of values. Then write an equation that models the relationship between the quantities.

   Sample answer: The distance Yasmine traveled in miles is equal to 50 times the number of hours she drove.

   \[ d = 50 \times t \text{ (multiplicative relationship).} \]

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (mi)</td>
<td>100</td>
<td>150</td>
<td>200</td>
<td>250</td>
</tr>
</tbody>
</table>

15. **Critical Thinking** Georgia knows that there is either an additive or multiplicative relationship between \( x \) and \( y \). She only knows a single pair of data values. Explain whether Georgia has enough information to write an equation that models the relationship between \( x \) and \( y \).

   No; with only one pair of values, Georgia cannot tell whether the relationship is additive or multiplicative, so she cannot write an equation for the relationship.

---

**EXTEND THE MATH**

**Activity** To introduce the idea of relationships that are not purely additive or multiplicative, have students find the \( y \)-values in the following tables. Remind them to use the order of operations. Have them compare and contrast these relationships with additive and multiplicative relationships.

1. \( y = 2x + 1 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
</tbody>
</table>

2. \( y = 3x - 2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td>10</td>
</tr>
</tbody>
</table>

3. \( y = \frac{1}{2}x + 3 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

4. \( y = \frac{1}{3}x - 1 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>
**LESSON 12.4 Representing Algebraic Relationships in Tables and Graphs**

**Common Core Standards**

The student is expected to:

- **Expressions and Equations—6.EE.9**

Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

**Mathematical Practices**

- **MP.4 Modeling**

**Engage**

**ESSENTIAL QUESTION**

*How can you use verbal descriptions, tables, and graphs to represent algebraic relationships?* Sample answer: You can make a table from the verbal description and then make a graph from the ordered pairs in the table. From a graph, you can make a table and write an equation.

**Motivate the Lesson**

*Ask:* Have you ever thought about running in a marathon? Do you know how many kilometers you could run in an hour? in two hours? Begin Explore Activity 1 to find out how to make a table and a graph to estimate how far you could run in a given period of time.

**Explore**

**EXPLORE ACTIVITY 1**

**Connect Multiple Representations**

Point out to students that the ordered pairs from each table are used to make the graphs. However, after the lines are drawn on the graphs, they represent a more complete picture of the relationship than the tables do because all nonnegative real numbers, not just whole numbers, are included in the graph.

**Explain**

**EXPLORE ACTIVITY 2**

**Focus on Math Connections**

Point out to students that when finding an equation from a graph, it is easier to first make a table of values from the graph. Then they can look for a pattern for the equation.

**Questioning Strategies**

- Is the relationship additive or multiplicative? Explain how you know. The relationship is additive, because the line drawn through the points does not go through the origin.
- Explain how you can find the entrance fee for the museum from the graph. The starting point of the graph is (0, 5). This ordered pair represents the cost of Cherise’s expenses at the museum without any purchases at the gift shop, so it represents the entrance fee, $5.

**Engage with the Whiteboard**

Ask a student volunteer to complete the table and identify the pattern. Then have the student write the equation to represent the total amount spent at the museum gift shop. Finally, discuss with the class what the independent and dependent variables are.
**EXPLORE ACTIVITY 1**

### Representing Algebraic Relationships

Angie's walking speed is 5 kilometers per hour, and May's is 4 kilometers per hour. Show how the distance each girl walks is related to time.

**A.** For each girl, make a table comparing time and distance.

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>Angie's distance (km)</th>
<th>May's distance (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>16</td>
</tr>
</tbody>
</table>

**B.** For each girl, make a graph showing her distance as it depends on time. Write an equation for each girl that relates distance to time.

- **Angie's graph:**
  - Distance: $y = 5x$
  - Graph: shows a line with a slope of 5,
  - Reflection: The girls can walk for fractional parts of an hour and can travel fractional parts of a kilometer.

- **May's graph:**
  - Distance: $y = 4x$
  - Graph: shows a line with a slope of 4,
  - Reflection: The girls can walk for fractional parts of an hour and can travel fractional parts of a kilometer.

**Reflect**

1. **Analyze Relationships** How can you use the tables to determine which girl is walking faster? How can you use the graphs? Use the tables to determine which girl is walking faster. Compare the distances walked after the same amount of time; compare the steepness of the lines.

**Math Background**

The equations in this lesson are linear equations. A linear equation is an equation whose solutions fall on a line on a coordinate plane. All solutions of a particular linear equation fall on the line, and all the points on the line are solutions of the equation. Linear equations have constant slope.

In the slope-intercept form, $y = mx + b$,
- $m$ is the slope and $b$ is the y-intercept.

Linear equations fit into the general category of polynomial equations. Linear equations are called first-degree equations, because the greatest power of $x$ is 1.
CHECK FOR UNDERSTANDING

Ask: Can the points on the graphs only have whole number coordinates? Explain your answer. No, the coordinates can be any pair of rational numbers that satisfies the equation. For example, (0.5, 1.5) is on the graph of \( y = x + 1 \). Limits on the values of the variables in equations are usually the result of real-world situations.

**Questioning Strategies**

- How can you plot points on the graph in A other than the ones in the table? Choose numbers on the \( x \)-axis other than the ones in the table. Add 1 to each value of \( x \), and plot the points \((x, y)\) that you produce.
- How can you plot points on the graph in B other than the ones in the table? Choose numbers on the \( x \)-axis other than the ones in the table. Multiply each value of \( x \) by 2, and plot the points \((x, y)\) that you produce.

**YOUR TURN**

**Avoid Common Errors**

Students may plot the points in the table but forget to draw a line connecting the points. Remind them that they must connect the points with a line for the graph to be correctly drawn.

**Elaborate**

**Talk About It**

**Summarize the Lesson**

Ask: How can you use tables and graphs to represent algebraic relationships? You can make a table from the verbal description and then make a graph from the ordered pairs in the table. From a graph, you can make a table and write an equation.

**GUIDED PRACTICE**

**Engage with the Whiteboard**

For Exercises 1–2, have students complete the table and graph the points on the coordinate grid on the whiteboard. Then have another student identify the pattern and write the equation.

**Avoid Common Errors**

**Exercises 1–2** Students may plot the points in the table but forget to draw a line connecting the points. Remind them that they must connect the points with a line for the graph to be drawn correctly.
Graphing an Equation

An ordered pair \((x, y)\) that makes an equation like \(y = x + 1\) true is called a solution of the equation. The graph of an equation represents all the ordered pairs that are solutions.

**EXAMPLE 1**

Graph each equation.

**A** \(y = x + 1\)

**STEP 1** Make a table of values. Choose some values for \(x\) and use the equation to find the corresponding values for \(y\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y = x + 1)</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 + 1 = 2</td>
<td>(1, 2)</td>
</tr>
<tr>
<td>2</td>
<td>2 + 1 = 3</td>
<td>(2, 3)</td>
</tr>
<tr>
<td>3</td>
<td>3 + 1 = 4</td>
<td>(3, 4)</td>
</tr>
<tr>
<td>4</td>
<td>4 + 1 = 5</td>
<td>(4, 5)</td>
</tr>
<tr>
<td>5</td>
<td>5 + 1 = 6</td>
<td>(5, 6)</td>
</tr>
</tbody>
</table>

**STEP 2** Plot the ordered pairs from the table.

**STEP 3** Draw a line through the plotted points to represent all of the ordered pair solutions of the equation.

**B** \(y = 2x\)

**STEP 1** Make a table of values. Choose some values for \(x\) and use the equation to find the corresponding values for \(y\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>(2x = y)</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 \times 1 = 2</td>
<td>(1, 2)</td>
</tr>
<tr>
<td>2</td>
<td>2 \times 2 = 4</td>
<td>(2, 4)</td>
</tr>
<tr>
<td>3</td>
<td>2 \times 3 = 6</td>
<td>(3, 6)</td>
</tr>
<tr>
<td>4</td>
<td>2 \times 4 = 8</td>
<td>(4, 8)</td>
</tr>
<tr>
<td>5</td>
<td>2 \times 5 = 10</td>
<td>(5, 10)</td>
</tr>
</tbody>
</table>

**STEP 2** Plot the ordered pairs from the table.

**STEP 3** Draw a line through the plotted points to represent all of the ordered pair solutions of the equation.

**Math Talk**

Is the ordered pair \((3.5, 4.5)\) a solution of the equation \(y = x + 1\)? Explain.

Yes; \((3.5, 4.5)\) is on the graph. You can substitute for the variables in the equation to check.

Frank mows lawns in the summer to earn extra money. He can mow 3 lawns every hour he works. (Explore Activity 1 and Explore Activity 2)

1. Make a table to show the relationship between the number of hours Frank works, \(x\), and the number of lawns he mows, \(y\). Graph the relationship and write an equation. Label the axes of your graph.

<table>
<thead>
<tr>
<th>Hours worked</th>
<th>Lawns mowed</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

   **Graph** \(y = 1.5x\). (Example 1)

2. Make a table to show the relationship.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.5</td>
<td>3</td>
<td>4.5</td>
<td></td>
</tr>
</tbody>
</table>

3. Plot the points and draw a line through them.

4. How can a table represent an algebraic relationship between two variables?

   **It shows pairs of values that satisfy the relationship.**

**DIFFERENTIATE INSTRUCTION**

**Cooperative Learning**

Have students work in pairs to write an equation with two variables. Each equation should involve addition. Collect students’ equations and randomly redistribute them. Have the students make tables for the equations and write solutions of the equations as ordered pairs. Then have the students graph the equations.

**Modeling**

Draw an equilateral triangle and a square, each with a side length of 6 inches, on the chalkboard. Have students find the perimeter of each. Ask students to come up with a formula for the perimeter of an equilateral triangle and a square. Then have students make a table and a graph for each formula.

**Additional Resources**

**Differentiated Instruction** includes:

- Reading Strategies
- Success for English Learners
- Reteach
- Challenge
12.4 LESSON QUIZ

The graph shows the number of bracelets Olivia can make in an hour.

1. Read the ordered pairs from the graph to make a table.
2. Write an equation to model the relationship.

The equation \( y = x + 2 \) represents the total cost of doing \( x \) loads of laundry at a laundromat in dollars, including buying a box of detergent.

3. Make a table that represents the relationship between number of loads and total cost.
4. Make a graph showing the relationship.

Answers

1. Number of hours 0 1 2
   Number of bracelets 0 4 8

2. \( y = 4x \)

3. Number of loads 0 1 2 3
   Total cost ($) 2 3 4 5

Evaluate

GUIDED AND INDEPENDENT PRACTICE

Concepts & Skills | Practice
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Explore Activity 1 | Exercise 1
Representing Algebraic Relationships
Explore Activity 2 | Exercises 1, 5–7
Writing an Equation from a Graph
Example 1 | Exercises 2–3, 8–9, 11
Graphing an Equation

Exercise | Depth of Knowledge (D.O.K.) | Mathematical Practices
--- | --- | ---
5–6 | 2 Skills/Concepts | MP.4 Modeling
7 | 3 Strategic Thinking | MP.7 Using Structure
8–9 | 2 Skills/Concepts | MP.2 Reasoning
10 | 3 Strategic Thinking | MP.3 Logic
11 | 3 Strategic Thinking | MP.7 Using Structure
12 | 3 Strategic Thinking | MP.3 Logic
13–14 | 3 Strategic Thinking | MP.7 Using Structure

Additional Resources

Differentiated Instruction includes:
- Leveled Practice worksheets

Exercise 11 combines concepts from the Common Core cluster "Represent and analyze quantitative relationships between dependent and independent variables."
12.4 Independent Practice

Students at Mills Middle School are required to work a certain number of community service hours. The table shows the numbers of additional hours several students worked beyond their required hours, as well as the total numbers of hours worked.

5. Read the ordered pairs from the graph to make a table.

<table>
<thead>
<tr>
<th>Additional hours</th>
<th>Total hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>15</td>
<td>35</td>
</tr>
<tr>
<td>20</td>
<td>40</td>
</tr>
</tbody>
</table>

6. Write an equation that expresses the total hours in terms of the additional hours.

\[ y = x + 20 \]

7. Analyze Relationships How many community service hours are students required to work? Explain.

20 hours; when 0 additional hours are worked, the total is 20 hours.

Beth is using a map. Let \( x \) represent a distance in centimeters on the map. To find an actual distance \( y \) in kilometers, Beth uses the equation \( y = \frac{x}{8} \).

8. Make a table comparing a distance on the map to the actual distance.

<table>
<thead>
<tr>
<th>Map distance (cm)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual distance (km)</td>
<td>8</td>
<td>16</td>
<td>24</td>
<td>32</td>
<td>40</td>
</tr>
</tbody>
</table>

9. Make a graph that compares the map distance to the actual distance. Label the axes of the graph.

10. Critical Thinking The actual distance between Town A and Town B is 64 kilometers. What is the distance on Beth’s map? Did you use the graph or the equation to find the answer? Why?

8 cm; sample answer: I used the equation because the scales on the graph don’t extend far enough.

EXTEND THE MATH PRE-AP

Activity Ask students if they know that you can find the approximate temperature by listening to snowy tree crickets chirp. The chirp rate of the cricket varies by temperature. The hotter it is, the more chirps per minute. The temperature in °F can be approximated by multiplying the number of chirps in one minute by \( \frac{1}{4} \) and adding 40. Write the equation that represents this situation. Then, make a table of values to approximate the temperature for 20, 40, 60, 80, and 100 chirps per minute.

\[ t = \frac{1}{4}c + 40 \]

where \( t \) is the Fahrenheit temperature and \( c \) is the number of chirps in a minute.

<table>
<thead>
<tr>
<th>( c )</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>45</td>
<td>50</td>
<td>55</td>
<td>60</td>
<td>65</td>
</tr>
</tbody>
</table>

11. Multistep The equation \( y = 9x \) represents the total cost \( y \) for \( x \) movie tickets. Label the axes of the graph.

a. Make a table and a graph to represent the relationship between \( x \) and \( y \).

<table>
<thead>
<tr>
<th>Number of tickets, ( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total cost ($), ( y )</td>
<td>9</td>
<td>18</td>
<td>27</td>
<td>36</td>
<td>45</td>
</tr>
</tbody>
</table>

b. Critical Thinking In this situation, which quantity is dependent and which is independent? Justify your answer.

Dependent: total cost; independent: number of tickets; the total cost depends on how many tickets were purchased.

c. Multiple Representations Eight friends want to go see a movie. Would you prefer to use an equation, a table, or a graph to find the cost of 8 movie tickets? Explain how you would use your chosen method to find the cost.

Sample answer: an equation; substitute 8 for \( x \) in \( y = 9x \) to get \( y = 9(8) = 72 \).

Focus on Higher Order Thinking

12. Critical Thinking Suppose you graph \( y = 5x \) and \( y = x + 500 \) on the same coordinate plane. Which line will be steeper? Why?

The graph of \( y = 5x \) will be steeper because \( y \) increases more rapidly for each value of \( x \).

13. Persevere in Problem Solving Marcus plotted the points \((0, 0), (6, 2), (18, 6), \) and \((21, 7)\) on a graph. He wrote an equation for the relationship. Find another ordered pair that could be a solution of Marcus’s equation. Justify your answer.

Sample answer: \((30, 10); \) every \( y \) value is \( \frac{1}{3} \) of the \( x \) value.

So, 30 = \( \frac{1}{3} \) (30).

14. Error Analysis The cost of a personal pizza is \$4. A drink costs \$1. Anna wrote the equation \( y = 4x + 1 \) to represent the relationship between total cost \( y \) of buying \( x \) meals that include one personal pizza and one drink. Describe Anna’s error and write the correct equation.

Anna’s equation does not show that every meal includes both a pizza and a drink; the correct equation is \( y = 5x \).

Work Area
Ready to Go On?

Assess Mastery
Use the assessment on this page to determine if students have mastered the concepts and standards covered in this module.

Response to Intervention

Intervention  Enrichment

Access Ready to Go On? assessment online, and receive instant scoring, feedback, and customized intervention or enrichment.

Online and Print Resources

Differentiated Instruction
• Reteach worksheets
• Challenge worksheets

ELL

Differentiated Instruction
• Success for English Learners

Pre-AP

ELL

Extend the Math Lesson Activities in TE

Additional Resources

Assessment Resources includes:
• Leveled Module Quizzes

Common Core Standards

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Exercises</th>
<th>Common Core Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.1</td>
<td>1–6</td>
<td>6.NS.6, 6.NS.6b, 6.NS.6c, 6.NS.8</td>
</tr>
<tr>
<td>12.2</td>
<td>7</td>
<td>6.EE.9</td>
</tr>
<tr>
<td>12.3</td>
<td>8–9</td>
<td>6.EE.9</td>
</tr>
<tr>
<td>12.4</td>
<td>10–11</td>
<td>6.EE.9</td>
</tr>
</tbody>
</table>
Assessment Readiness

Assessment Readiness Tip Some items are called context-based items, which means the student has to examine each answer choice in order to determine the correct answer.

Item 2 If students don’t remember that for every point in quadrant II the x-coordinate is a negative number and the y-coordinate is a positive number, they may need to plot each point to see that choice C is the correct answer.

Item 5 To find the point that the graph of \( y = 10 + x \) does not pass through, students may need to graph each point on a coordinate grid to see that choice C is the correct answer.

Avoid Common Errors

Item 1 Students may forget what the first and second numbers in an ordered pair mean. Remind students that the first number is the x-coordinate, and the second is the y-coordinate.

Item 4 Students often will get the independent and dependent quantities backward in problems, thereby choosing A for the answer. Remind students that the dependent quantity depends on the independent quantity. Therefore, the number of points earned depends on the number of prizes captured.

Additional Resources

Common Core Standards

<table>
<thead>
<tr>
<th>Items</th>
<th>Grade 6 Standards</th>
<th>Mathematical Practices</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.NS.6c</td>
<td>MP.2</td>
</tr>
<tr>
<td>2</td>
<td>6.NS.6b</td>
<td>MP.7</td>
</tr>
<tr>
<td>3*</td>
<td>6.EE.2a, 6.EE.9</td>
<td>MP.4</td>
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<td>4</td>
<td>6.EE.9</td>
<td>MP.4</td>
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<td>5*</td>
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<tr>
<td>6</td>
<td>6.EE.9</td>
<td>MP.2, MP.4</td>
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<tr>
<td>7</td>
<td>6.NS.8, 6.NS.6c</td>
<td>MP.4, MP.7</td>
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</table>

* Item integrates mixed review concepts from previous modules or a previous course.