Getting Ready for California Common Core Standards

The topics described in the Standards for Mathematical Content will vary from year to year. However, the *way* in which you learn, study, and think about mathematics will not. The Standards for Mathematical Practice describe skills that you will use in all of your math courses. These pages show some features of your book that will help you gain these skills and use them to master this year's topics.

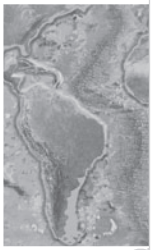
MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem… They analyze givens, constraints, relationships, and goals. They make conjectures about the form… of the solution and plan a solution pathway…

Explain 3 **Modeling Using a Function Graph**

The domain of a real-world situation may have to be limited in order to have reasonable answers. Only no numbers can be used to represent quantities such as time, distance, and the number of people. When both and the range of a function are limited to nonnegative values, the function is graphed only in Quadrant I.

The Mid-Atlantic Ridge separates the North and South American Plates from the Eurasian and African Plates. The function $y = 2.5x$ relates the number of centimeters y the Mid-Atlantic Ridge spreads after x years. Graph the function and use the graph to estimate how many centimeters the Mid-Atlantic Ridge spreads in 4.5 years.



Analyze Information

Identify the important information:

- The function _____ describes how many centimeters the Mid-Atlantic

Problem-solving examples and exercises lead students through problem solving steps.

MP.2 Reason abstractly and quantitatively.

Mathematically proficient students… bring two complementary abilities to bear on problems…: the ability to decontextualize—to abstract a given situation and represent it symbolically… and the ability to contextualize, to pause… in order to probe into the referents for the symbols involved.

H.O.T. Focus on Higher Order Thinking

MODULE PERFORMANCE TASK

Who Wins the Race?

Focus on Higher Order Thinking exercises in every lesson and **Performance Tasks** in every unit require you to use logical reasoning, represent situations symbolically, use mathematical models to solve problems, and state your answers in terms of a problem context.

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MP.3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students… justify their conclusions, [and]… distinguish correct… reasoning from that which is flawed.

Reflect

5. What is the meaning of the y-intercepts for the functions $A(t$

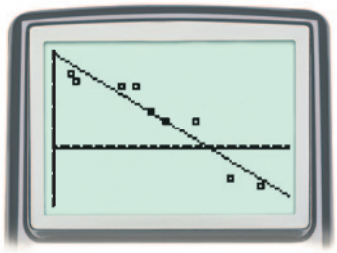
Essential Question-Check-In How can you compare a linear function represented in a table to one represented as a graph?

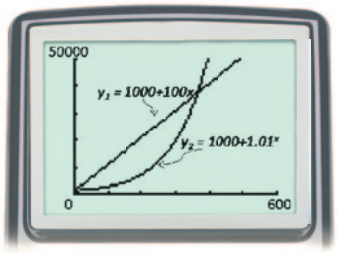
Essential Question Check-in and **Reflect** in every lesson ask you to evaluate statements, explain relationships, apply mathematical principles, make conjectures, construct arguments, and justify your reasoning.

MP.4 Model with mathematics.

Mathematically proficient students can apply… mathematics… to… problems… in everyday life, society, and the workplace.

Explain 2

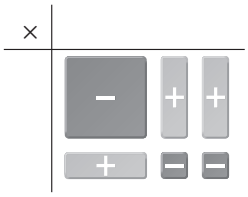


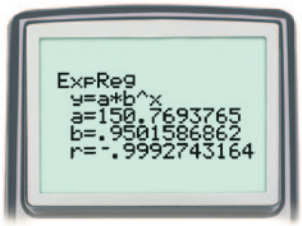


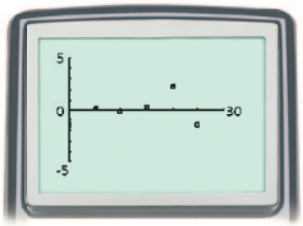
Real-world examples and **mathematical modeling** apply mathematics to other disciplines and real-world contexts such as science and business.

MP.5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a… problem… [and] are… able to use technological tools to explore and deepen their understanding…







Exploration Activities in lessons use concrete and technological tools, such as manipulatives or graphing calculators, to explore mathematical concepts.

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MP.6 Attend to precision.

Mathematically proficient students… communicate precisely… with others and in their own reasoning… [They] give carefully formulated explanations…

Reflect

1. **Communicate Mathematical Ideas** What are the limitatio

3. **Discussion** The last step of the solution process seems to impl all values of b? Justify your answer.

Precision refers not only to the correctness of calculations but also to the proper use of mathematical language and symbols. **Communicate Mathematical Ideas** exercises and **Discussion** question help you learn and use the language of math to communicate mathematics precisely.

MP.7 Look for and make use of structure.

Mathematically proficient students… look closely to discern a pattern or structure… They can also step back for an overview and shift perspectives.

How the Value of m Changes	Effect on the Graph of $f(x) = mx$
A. Increase m when $m > 1$.	_____ Graph becomes steeper.
B. Decrease m when $0 < m < 1$.	_____ Graph becomes less steep.
C. Decrease m when $m < -1$.	
D. Increase m when $0 > m > -1$.	

$$(2x + 5)(3x + 2) = \underbrace{6x^2}_{\text{F}} + \underbrace{4x}_{\text{O}} + \underbrace{15x}_{\text{I}} + \underbrace{10}_{\text{L}} = 6x^2 + 19x + 10$$

F The sum of the coefficients of the first terms is a.

O, I The sum of the coefficients of the outer and inner products is b.

L The product of the last terms is c.

Throughout the lessons, you will observe regularity in mathematical structures in order to make generalizations and make connections between related problems. For example, you can see the same structure used when learning to multiply binomials and factor trinomials.

MP.8 Look for and express regularity in repeated reasoning.

Mathematically proficient students… look both for general methods and for shortcuts… [and] maintain oversight of the process, while attending to the details.

Arithmetic: 3, 5, 7, …		
Term Number	Term	Difference
1	3	—
2	5	5 − 3 = ☐
3	7	7 − 5 = ☐
4	9	9 − 7 = ☐
5	11	11 − 9 = ☐

Explore 1 **Exploring Growth Patterns of Geometric Sequences**

Examples in your book group similar types of problems together to allow you to look for patterns and make generalizations.

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CA CC **N-Q.1**

Use units as a way to understand problems and to guide the solution of multi-step problems choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

Key Vocabulary

unit analysis/ dimensional analysis *(analisis dimensional)*
A practice of converting measurements and checking among computed measurements.

What It Means to You

Units can help you confirm that you are on the right track when solving a problem. If your units work out right, your answer is probably right.

EXAMPLE N-Q.1

Li's car gets 40 miles per gallon of gas. At this rate, she can go 420 miles on a full tank. She has driven 245 miles on the current tank. How many gallons of gas are left in the tank?

STEP 1 Find the number of miles remaining.

$$420 \text{ mi} - 245 \text{ mi} = 175 \text{ mi}$$

STEP 2 Find the number of gallons.

$$x = 175 \text{ mi} \cdot \frac{\text{gal}}{40 \text{ mi}} = \frac{175 \cancel{\text{mi}}}{40 \cancel{\text{mi}}} \text{gal} = 4.375 \text{ gal}$$

According to these measurements there are 4.375 gallons left in the tank.

CA CC **N-Q.2**

Define appropriate quantities for the purpose of descriptive modeling.

What It Means to You

Defining quantities carefully helps guide your problem solving when you model a situation.

EXAMPLE N-Q.2

To find all sets of 3 consecutive even numbers with sums from 60 to 70, let x be any integer. Then $2x$ is an even integer, and $2x + 2$ and $2x + 4$ are the next consecutive even integers.

$$\begin{aligned} 60 &\leq 2x + (2x + 2) + (2x + 4) \leq 70 \\ 60 &\leq 6x + 6 \leq 70 \\ 54 &\leq \quad 6x \quad \leq 64 \\ 9 &\leq \quad x \quad \leq 10\tfrac{2}{3} \end{aligned}$$

Because x can be 9 or 10, the first even integer is $2(9) = 18$ or $2(10) = 20$. The sets are 18, 20, 22 or 20, 22, 24.

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CA CC **N-Q.3**

Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Key Vocabulary

accuracy *(exactitud)*
The closeness of a given measurement to the actual measurement.

What It Means to You

You will learn to use precision and significant digits when calculating with measurements.

EXAMPLE N-Q.3

Which is the more precise measurement: 5.7 m or 568 cm?
Which is the more accurate measurement: 5.7 m or 568 cm?

	Precision	Accuracy
5.7 m	5.7 m = 570 cm is measured to nearest *ten* centimeters.	5.7 has 2 significant digits.
568 cm	568 cm is measured to the nearest centimeter.	568 has 3 significant digits.

568 cm is more precise because it is measured to a smaller unit.
568 cm is more accurate because it has more significant digits.

CA CC **A-SSE.1a**

Interpret parts of an expression, such as terms, factors, and coefficients.

Key Vocabulary

coefficient *(coeficiente)*
A number that is multiplied by a variable.

constant *(constante)*
A value that does not change.

expression *(expresión)*
A mathematical phrase that contains operations, numbers, and/or variables.

factor *(factor)*
A number or expression that is multiplied by another number or expression to get a product.

term of an expression *(término de una expresión)*
The parts of the expression that are added or subtracted.

What It Means to You

You will interpret parts of an expression in a particular real-world context.

EXAMPLE A-SSE.1a

Ann's monthly phone bill can be modeled by the expression $0.10x + 30$, where x represents the number of long-distance minutes used.

What is the coefficient? What does it represent?

> 0.10 is the coefficient. It represents the cost of each long-distance minute in dollars.

What is the constant? What does it represent?

> 30 is the constant. It represents the cost in dollars of phone service when no long-distance minutes are used.

How many terms are there in the expression? What are they?

> There are two terms, $0.10x$ and 30.

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CA CC A-SSE.1b

Interpret complicated expressions by viewing one or more of their parts as a single entity.

Key Vocabulary

expression *(expresión)*
A mathematical phrase that contains operations, numbers, and/or variables.

What It Means to You

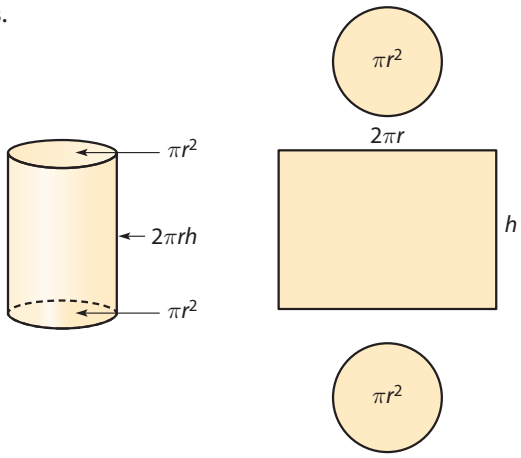
You will analyze different parts of an expression and discover their meaning.

EXAMPLE A-SSE.1b

The surface area of a cylinder is equal to $2\pi rh + 2\pi r^2$. Describe what each term in the formula represents.

$2\pi rh$ is equivalent to the circumference of a circle times the height of the cylinder. It represents the lateral surface area.

$2\pi r^2$ is equivalent to twice the area of a circle. It represents the total area of the circular bases.



CA CC A-CED.1

Create equations and inequalities in one variable including ones with absolute value and use them to solve problems.

Key Vocabulary

equation *(ecuación)*
A mathematical statement that two expressions are equivalent.

variable *(variable)*
a symbol used to represent a quantity that can change

What It Means For You

You can write an equation to represent a real-world problem and then use algebra to solve the equation and find the answer.

EXAMPLE A-CED.1

Michael is saving money to buy a trumpet. The trumpet costs \$670. He has \$350 saved, and each week he adds \$20 to his savings. How long will it take him to save enough money to buy the trumpet?

$$\text{cost of trumpet} = \text{current savings} + \text{additional savings}$$

$$670 = 350 + 20w$$

$$320 = 20w$$

$$16 = w$$

It will take Michael 16 weeks to save enough money.

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CA CC A-CED.2

Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Key Vocabulary

equation *(ecuación)*
A mathematical statement that two expressions are equivalent.

What It Means to You

Creating equations in two variables to describe relationships gives you access to the tools of graphing and algebra to solve the equations.

EXAMPLE A-CED.2

A customer spent \$29 on wristbands and gel watches. Wristbands cost \$2.50 each and gel watches cost \$1.75 each.

w = number of wristbands bought

g = number of gel watches bought

$2.5w + 1.75g = 29$

CA CC A-CED.3

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

Key Vocabulary

inequality *(desigualdad)*
A statement that compares two expressions by using one of the following signs: $<$, $>$, $\leq$, $\geq$, or $\neq$.

solution of an inequality in one variable *(solución de una desigualdad en una variable)*
A value or values that make the inequality true.

What It Means to You

You can use inequalities to represent limits on the values in a situation so that the solutions make sense in a real-world context.

EXAMPLE A-CED.3

Anyone riding the large water slide at a the park must be at least 40 inches tall.

Let h represent the heights that are allowed.

Height is at least 40 inches.

$h \geq 40$

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CA CC A-CED.4

Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Key Vocabulary

formula *(fórmula)*
A literal equation that states a rule for a relationship among quantities.

What It Means to You

You will solve a formula for a particular variable.

EXAMPLE A-CED.4

Write the formula for the area of a trapezoid, $A = \frac{1}{2}(b_1 + b_2)h$, in terms of h.

$2A = 2\left(\frac{1}{2}(b_1 + b_2)h\right)$	Multiply both sides of the equation by 2.
$\frac{2A}{(b_1 + b_2)} = \frac{(b_1 + b_2)h}{(b_1 + b_2)}$	Divide both sides of the equation by $(b_1 + b_2)$.
$\frac{2A}{(b_1 + b_2)} = h$	Simplify the right side.

CA CC A-REI.1

Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Key Vocabulary

equation *(ecuación)*
A mathematical statement that two expressions are equivalent.

What It Means to You

You will show how a simple equation can be solved using the properties of equality.

EXAMPLE A-REI.1

Solve $2x + 3 = 11$. Justify each step in the solution process.

$2x + 3 - 3 = 11 - 3$	Subtraction Property of Equality
$2x = 8$	Simplify.
$\frac{2x}{2} = \frac{8}{2}$	Division Property of Equality
$x = 4$	Simplify.

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CA CC A-REI.3

Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Key Vocabulary

linear inequality in one variable *(desigualdad lineal en una variable)*
An inequality that can be written in one of the following forms; $ax < b$, $ax > b$, $ax \leq b$, $ax \geq b$, or $ax \neq b$, where a and b are constants and $a \neq 0$.

What It Means to You

Being able to solve inequalities lets you answer questions where a range of solutions is possible.

EXAMPLE A-REI.3

Solve the inequality for t to find what grades on the final exam will give Cleo a course grade of "A".

$705 + 2t \geq 895$	Cleo has 705 points and needs at least 895.
$2t \geq 190$	Subtract 705 from both sides.
$t \geq 95$	Divide both sides by 2.

Cleo needs to earn a 95 or above on the final exam.

CA CC A-REI.3.1

Solve one-variable equations and inequalities involving absolute value, graphing the solutions and interpreting them in context.

Key Vocabulary

absolute value equation *(ecuación de valor absoluto)*
An equation that contains absolute-value expressions.

What It Means to You

You will solve equations involving absolute value.

EXAMPLE A-REI.3.1

Solve $4|x + 2| = 24$.

$$\frac{4|x+2|}{4} = \frac{24}{4}$$ Divide both sides by 4.

$$|x + 2| = 6$$ Think: What numbers are 6 units from 0?

Case 1	Case 2	
$x + 2 = -6$	$x + 2 = 6$	Rewrite the equation as two cases.
$\underline{\quad -2 \quad -2}$	$\underline{\quad -2 \quad -2}$	Subtract 2 from both sides.
$x = -8$	$x = 4$	

The solutions are -8 and 4.

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CA CC A-REI.5

Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

Key Vocabulary

elimination method *(eliminación)*
A method used to solve systems of equations in which one variable is eliminated by adding or subtracting two equations of the system.

system of linear equations *(sistema de ecuaciones lineales)*
A system of equations in which all of the equations are linear.

What It Means to You

You will solve a system of linear equations by using elimination.

EXAMPLE A-REI.5

Solve $\begin{cases} 4x + 3y = 2 \\ -2x + 5y = 12 \end{cases}$ using elimination.

$2(-2x + 5y) = 2(12)$ — Multiply the second equation by 2.

$-4x + 10y = 24$ — Simplify.

$$\begin{array}{r} 4x + 3y = 2 \\ +(-4x + 10y) = +24 \\ \hline y = \frac{26}{13} \\ y = 2 \end{array}$$

Add the new equation to the first equation.

Solve.

Substitute the y-value of 2 into one of the original equations:

$$\begin{aligned} 4x + 3(2) &= 2 \\ 4x + 6 &= 2 \\ 4x &= -4 \\ x &= -1 \end{aligned}$$

The solution of the system is $(-1, 2)$.

CA CC A-REI.6

Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Key Vocabulary

system of linear equations *(sistema de ecuaciones lineales)*
A system of equations in which all of the equations are linear.

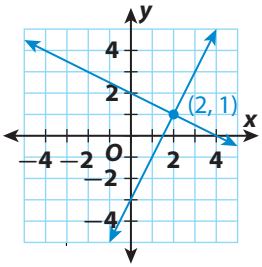
What It Means to You

You can solve systems of equations to find out when two relationships involving the same variables are true at the same time.

EXAMPLE A-REI.6

Find the solution of the system of equations.

$$\begin{cases} y = 2x - 3 \\ y = -\frac{1}{2}x + 2 \end{cases}$$



The solution of the system is the point (x, y) that satisfies both equations simultaneously. On a graph, it is the point at which the lines intersect, $(2, 1)$.

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CA CC A-REI.10

Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Key Vocabulary

solution of an equation in two variables *(solución de una ecuación en dos variables)*
An ordered pair of values for the variables making the equation true.

What It Means to You

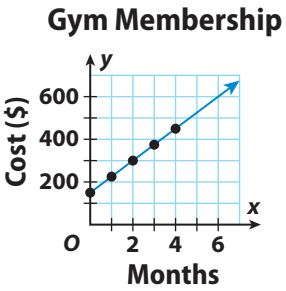
You can represent mathematical relationships with words, equations, tables, and graphs.

EXAMPLE A-REI.10

Membership costs \$150 plus \$75 per month.

$y = 75x + 150$

Months	Cost (\$)
0	150
1	225
2	300
3	375
4	450



CA CC A-REI.11

Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Key Vocabulary

***x*-coordinate** *(coordenada x)*
The first number in an ordered pair, which indicates the horizontal distance of a point from the origin on the coordinate plane.

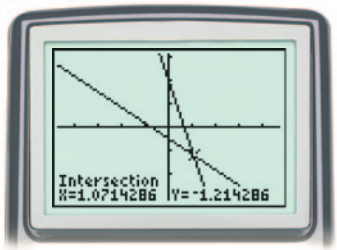
What It Means to You

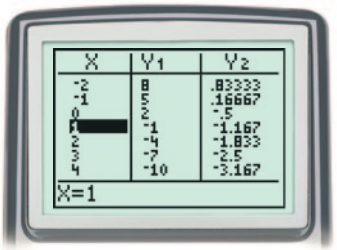
You will solve an equation in one variable by graphing a system of linear equations.

EXAMPLE A-REI.11

Solve $-3x + 2 = -\frac{2}{3}x - \frac{1}{2}$ by using technology.

Graph the corresponding functions $y = -3x + 2$ and $y = -\frac{2}{3}x - \frac{1}{2}$ on a graphing calculator. The x-coordinate of the intersection point, which is approximately 1, is the solution to the one-variable equation $-3x + 2 = -\frac{2}{3}x - \frac{1}{2}$.





So, when $x = 1$ is substituted into each side of the equation, each side simplifies to a value close to -1, as shown in the table.

This same solution approach can be used whether the functions representing each side of the original equation are linear, nonlinear, or both.

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CA CC A-REI.12

Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Key Vocabulary

half-plane *(semiplano)*
The part of the coordinate plane on one side of a line, which may include the line.

solution of an inequality in two variables *(solución de una desigualdad en dos variables)*
An ordered pair or ordered pairs that make the inequality true.

system of linear inequalities *(sistema de desigualdades lineales)*
A system of inequalities in which all of the inequalities are linear.

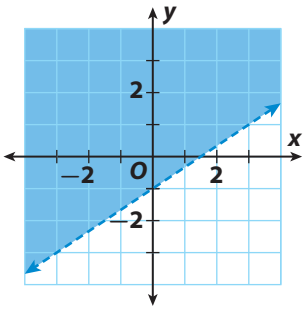
What It Means to You

Systems of linear inequalities model many real-life situations where you want to know when one or more conditions are met, but where there are many possible solutions.

EXAMPLE A-REI.12

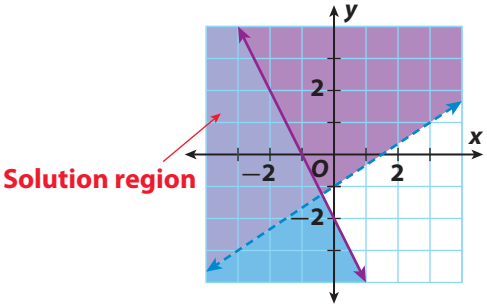
$y > \frac{2}{3}x - 1$

Linear Inequality



$$\begin{cases} y > \frac{2}{3}x - 1 \\ y \leq -2x - 2 \end{cases}$$

System of Two Linear Inequalities



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CA CC F-IF.1

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

Key Vocabulary

domain *(diminio)*
The set of all possible input values of a function.

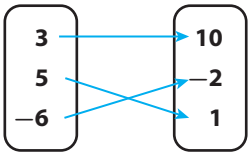
range *(recorrido o rango)*
The set of all possible output values of a function.

What It Means to You

A function model guarantees that for any input value, you will get a unique output value.

EXAMPLE F-IF.1

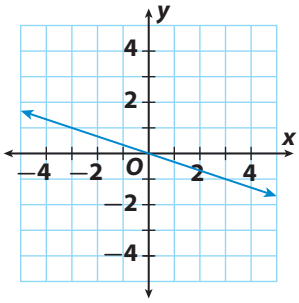
Categorize each of the following as a function or not a function.



A function: each input number in the left box is matched with exactly one output number in the right box.

$\{(0, 3), (2, -1), (0, 0)\}$

Not a function: the input 0 is matched to two outputs (3 and 0).



A function: each input (x-value) is matched with exactly one output (y-value).

CA CC F-IF.2

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Key Vocabulary

function notation *(notación de función)*
If x is the independent variable and y is the dependent variable, then the function notation for y is $f(x)$, read "f of x," where f names the function.

What It Means to You

You will evaluate a function for a given input value.

EXAMPLE F-IF.2

If $g(x) = 2x^2 - 3x + 6$, find $g(-2)$.

$$g(x) = 2x^2 - 3x + 6$$

$$g(-2) = 2(-2)^2 - 3(-2) + 6$$ Substitute -2 for each value of x.

$$g(-2) = 20$$ Simplify.

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Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.

Key Vocabulary

recursive rule for *n*th term of a sequence *(fórmula recurrente para hallar el enésimo término de una sucesión)*
A rule for a sequence in which one or more previous terms are used to generate the next term.

sequence *(sucesión)*
A list of numbers that often form a pattern.

What It Means to You

You will write a recursive rule for a sequence.

EXAMPLE F-IF.3

Write a recursive rule for the sum of the numbers found in each row of Pascal's triangle.

Row 0						**1**					
Row 1					**1**		**1**				
Row 2				**1**		**2**		**1**			
Row 3			**1**		**3**		**3**		**1**		
Row 4		**1**		**4**		**6**		**4**		**1**	
Row 5	**1**		**5**		**10**		**10**		**5**		**1**

Row 0 has a sum of 1, Row 1 has a sum of 2, Row 3 has a sum of 4, Row 4 has a sum of 8, and so on. The sum of each row is twice the sum of the row before. So, the recursive rule is:

$$f(0) = 1, f(n+1) = 2 \cdot f(n) \text{ for } n \geq 0$$

CA CC F-IF.4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Key Vocabulary

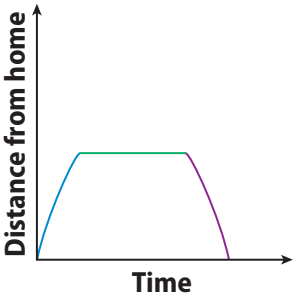
slope *(pendiente)*
The slope of a line is the ratio of rise to run for any two points on the line.

What It Means to You

Learning to interpret a graph enables a deep visual understanding of all sorts of relationships.

EXAMPLE F-IF.4

A group of friends walked to the town market, did some shopping there, then returned home.



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Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

Key Vocabulary

domain *(dominio)*
The set of all first coordinates (or *x*-values) of a relation or function.

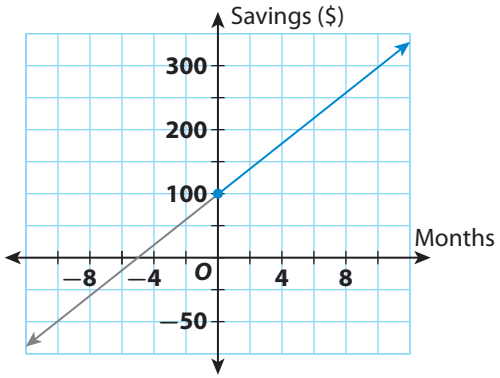
function *(función)*
A relation in which every domain value is paired with exactly one range value.

What It Means to You

You will identify an appropriate domain for a function.

EXAMPLE F-IF.5

Sarah opens a savings account with a deposit of $100 and saves an additional $20 per month. Sarah's total savings is modeled by the graphed function. What is an appropriate domain for the function?



An appropriate domain is all real numbers greater than or equal to 0.

CA CC F-IF.6

Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Key Vocabulary

rate of change *(tasa de cambio)*
A ratio that compares the amount of change in a dependent variable to the amount of change in an independent variable.

What It Means to You

Average rate of change measures the change in the dependent variable over the change in the independent variable. This helps you understand how quickly the values in a function change.

EXAMPLE F-IF.6

$$\text{Average rate of change} = \frac{180 - 60}{3 - 1} = 60 \text{ mi/h}$$

Time (hours)	1	2	3	4
Distance (miles)	60	120	180	240

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CA CC F-IF.7a

Graph linear and quadratic functions and show intercepts, maxima, and minima.

Key Vocabulary

quadratic function *(función cuadrática)*
A function that can be written in the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$.

***x*-intercept** *(intersección con el eje x)*
The x-coordinate(s) of the point(s) where a graph intersects the x-axis.

***y*-intercept** *(intersección con el eje y)*
The y-coordinate(s) of the point(s) where a graph intersects the y-axis.

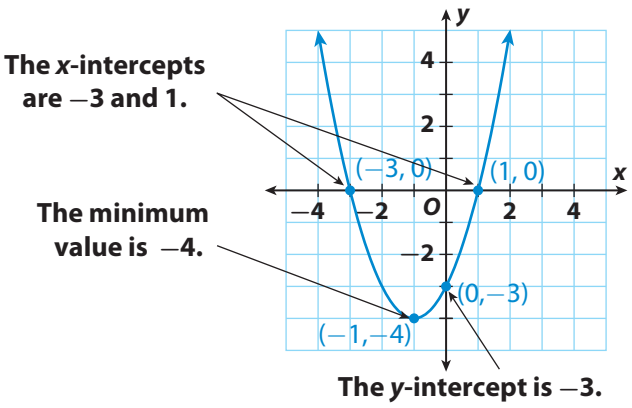
maximum/minimum value of a function *(máximo/mínimo de una función)*
The y-value of the highest/lowest point on the graph of a function.

What It Means to You

The graph of a quadratic function has key features that are helpful when interpreting a real-world quadratic model: the intercepts and the maximum or minimum value.

EXAMPLE F-IF.7a

Graph of $y = x^2 + 2x - 3$ and identify its intercepts and maximum or minimum value.



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CA CC **F-IF.7e**

Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Key Vocabulary

exponential function *(function exponencial)*
A function that can be written in the form $f(x) = ab^x$.

What It Means to You

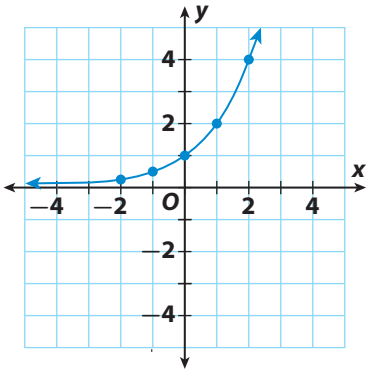
You will learn to graph a new type of function, called an exponential function, in which successive output values have a constant ratio for each unit increase in the input values.

EXAMPLE F-IF.7e

Graph the function $f(x) = 2^x$.

Find points on the graph using a table, then graph the points and connect them with a smooth curve.

x	$f(x)$
−2	0.25
−1	0.5
0	1
1	2
2	4



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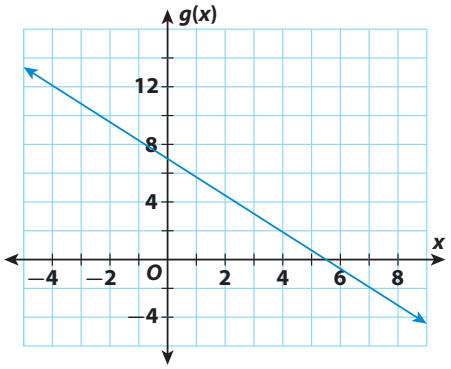
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

What It Means to You

You will find the x- and y-intercepts for a function represented by an equation and a function represented by a graph.

EXAMPLE F-IF.9

Find the x- and y-intercepts for the functions $f(x) = 6x - 3$ and $g(x)$ as shown on the grid.



To find the intercepts for $f(x)$, find the solutions when $f(x) = 0$ and when $x = 0$.

$$\begin{aligned} f(x) &= 6x - 3 \\ 0 &= 6x - 3 \\ 3 &= 6x \\ x &= \tfrac{1}{2} \end{aligned} \qquad \begin{aligned} f(x) &= 6x - 3 \\ f(x) &= 6(0) - 3 \\ f(x) &= -3 \end{aligned}$$

The x-intercept is $\frac{1}{2}$, and the y-intercept is -3.

To find the intercepts for $g(x)$, find where the graph crosses the x-axis and the y-axis. The x-intercept is 5.5, and the y-intercept is 7.

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Determine an explicit expression, a recursive process, or steps for calculation from a context.

What It Means to You

Real-world situations frequently involve patterns that you can model by sequences or sums of their terms. Translating a pattern into mathematical language helps you to solve problems with it.

EXAMPLE F-BF.1a

The table gives the number of cards in each row (starting at the top) of a simple triangular house of cards. Identify the sequence.

Row	1	2	3
Diagram			
Cards	2	5	8

The number of cards increases by 2 with each row: it is 1 less than 3 times the row number. The sequence is $a_n = 3n - 1$.

CA CC F-BF.1b

Combine standard function types using arithmetic operations.

Key Vocabulary

linear function *(función lineal)*
A function that can be written in the form $y = mx + b$, where x is the independent variable and m and b are real numbers. Its graph is a line.

What It Means to You

You will add and subtract linear functions.

EXAMPLE F-BF.1b

Given the functions $f(x) = 3x + 8$ and $g(x) = 6x - 5$, find $h(x) = f(x) + g(x)$ and $j(x) = f(x) - g(x)$.

When adding or subtracting functions, you can use the Commutative and Associative Properties to help you group like terms.

$$\begin{aligned} h(x) &= f(x) + g(x) \\ &= (3x + 8) + (6x - 5) \\ &= (3x + 6x) + (8 - 5) \\ &= 9x + 3 \end{aligned}$$

$$\begin{aligned} j(x) &= f(x) - g(x) \\ &= (3x + 8) - (6x - 5) \\ &= (3x - 6x) + (8 + 5) \\ &= -3x + 13 \end{aligned}$$

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CA CC F-BF.2

Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Key Vocabulary

arithmetic sequence *(sucesión aritmética)*
A sequence whose successive terms differ by the same nonzero number d, called the *common difference*.

geometric sequence *(sucesión geométrica)*
A sequence in which the ratio of successive terms is a constant r, called the *common ratio*, where $r \neq 0$ and $r \neq 1$.

What It Means to You

You will write a recursive rule and an explicit rule for arithmetic and geometric sequences.

EXAMPLE F-BF.2

Consider the arithmetic sequence 6, 9, 12, 15, 18, …

Write the recursive rule.

Given $f(1), f(n) = f(n-1) + d$ for $n \geq 2$ — Write the general form.

$f(1) = 6, f(n) = f(n-1) + 3$ for $n \geq 2$ — Use $f(1) = 6$. Substitute 3 for d.

The recursive rule is $f(1) = 6, f(n) = f(n-1) + 3$ for $n \geq 2$.

Write the explicit rule.

$f(n) = f(1) + d(n-1)$ — Write the general form.

$f(n) = 6 + 3(n-1)$ — Substitute 6 for $f(1)$ and 3 for d.

The explicit rule is $f(n) = 6 + 3(n-1)$.

CA CC F-BF.3

Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Key Vocabulary

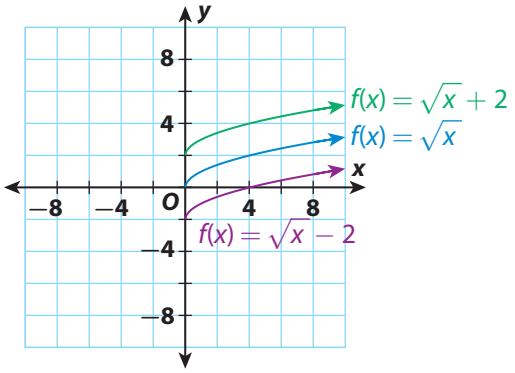
function notation *(notación de función)*
The notation used to describe a function.

What It Means to You

You can change a function by adding or multiplying by a constant. The result will be a new function that is a transformation of the original function.

EXAMPLE F-BF.3

The graph below shows examples of vertical translation of the function $f(x) = \sqrt{x}$ which occurs when a constant is added or subtracted to the function.



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CA CC **F-LE.1a**

Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

Key Vocabulary

linear function *(función lineal)*
A function that can be written in the form $y = mx + b$, where x is the independent variable and m and b are real numbers. Its graph is a line.

exponential function *(función exponencial)*
A function of the form $f(x) = ab^x$, where a and b are real numbers with $a \neq 0$, $b > 0$, and $b \neq 1$.

What It Means to You

You will show that linear functions change by equal differences, while exponential functions change by equal factors.

EXAMPLE F-LE.1a

Prove that linear functions change by equal differences, $f(x_2) - f(x_1) = f(x_4) - f(x_3)$, over equal intervals, $x_2 - x_1 = x_4 - x_3$.

Prove that exponential functions change by equal factors, $\frac{g(x_2)}{g(x_1)} = \frac{g(x_4)}{g(x_3)}$, over equal intervals, $x_2 - x_1 = x_4 - x_3$.

Linear Function

Given: $x_2 - x_1 = x_4 - x_3$
f is of the form $f(x) = mx + b$

Prove: $f(x_2) - f(x_1) = f(x_4) - f(x_3)$

$$x_2 - x_1 = x_4 - x_3$$

$$m(x_2 - x_1) = m(x_4 - x_3)$$

$$mx_2 - mx_1 = mx_4 - mx_3$$

$$mx_2 + b - mx_1 - b = mx_4 + b - mx_3 - b$$

$$(mx_2 + b) - (mx_1 + b) = (mx_4 + b) - (mx_3 + b)$$

$$f(x_2) - f(x_1) = f(x_4) - f(x_3)$$

Exponential Function

Given: $x_2 - x_1 = x_4 - x_3$
g is of the form $g(x) = ab^x$ where $b \neq 0$

Prove: $\frac{g(x_2)}{g(x_1)} = \frac{g(x_4)}{g(x_3)}$

$$x_2 - x_1 = x_4 - x_3$$

$$b^{(x_2 - x_1)} = b^{(x_4 - x_3)} \qquad \textit{if } x = y, \textit{ then } b^x = b^y.$$

$$\frac{b^{x_2}}{b^{x_1}} = \frac{b^{x_4}}{b^{x_3}} \qquad b^{x-y} = \frac{b^x}{b^y}$$

$$\frac{ab^{x_2}}{ab^{x_1}} = \frac{ab^{x_4}}{ab^{x_3}}$$

$$\frac{g(x_2)}{g(x_1)} = \frac{g(x_4)}{g(x_3)}$$

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CA CC **F-LE.1b**

Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

Key Vocabulary

linear function *(función lineal)*
A function that can be written in the form $y = mx + b$, where x is the independent variable and m and b are real numbers. Its graph is a line.

exponential function *(función exponencial)*
A function of the form $f(x) = ab^x$, where a and b are real numbers with $a \neq 0$, $b > 0$, and $b \neq 1$.

What It Means to You

A linear function models a *constant amount* of change for equal intervals, while an exponential function models a *constant factor* of growth for equal intervals.

EXAMPLE F-LE.1b

Which of the following tables shows linear growth? Which models exponential growth?

Mass (kg)	4	5	6	7	8	9	10
Length (cm)	30.6	32	33.4	34.8	36.2	37.6	39

+1.4 +1.4 +1.4 +1.4 +1.4 +1.4

For each additional kilogram, the length increases by an amount of 1.4 centimeters. The table above shows linear growth.

Time (yr)	5	6	7	8	9	10
Buffalo	124	150	185	213	261	322

×1.2 ×1.2 ×1.2 ×1.2 ×1.2

For each additional year, the number of buffalo increases by a factor (to the nearest tenth) of 1.2. The table above shows exponential growth.

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CA CC F-LE.1c

Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Key Vocabulary

exponential function *(función exponencial)*
A function of the form $f(x) = ab^x$, where a and b are real numbers with $a \neq 0$, $b > 0$, and $b \neq 1$.

What It Means to You

An exponential function models a *constant factor*, or *constant ratio* of change for equal intervals.

EXAMPLE F-LE.1c

Which of the tables shows data that could be modeled by an exponential function?

Value of a car		
Car's Age (yr)	**Value ($)**	
0	20,000	
1	17,000	$\times$ 0.85
2	14,450	$\times$ 0.85
3	12,282.50	$\times$ 0.85

+1, +1, +1

Ratio is constant.

Since the ratio is constant, the data above can be modeled by an exponential function.

Height of Bridge Suspension		
Cable's Distance from Tower (ft)	**Cable's Height (ft)**	
0	400	
100	256	−144, $\times$ 0.64
200	144	−112, $\times$ 0.56
300	64	−80, $\times$ 0.44

+100, +100, +100

The ratio is not constant. The data above cannot be modeled by an exponential function.

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Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

What It Means to You

You can construct a model of a linear or exponential function from different descriptions or displays of the same situation.

EXAMPLE F-LE.2

A ball is dropped 81 inches onto a hard surface. The table shows the ball's height on successive bounces. Write a model for the height reached as a function of the number of bounces.

Bounce	1	2	3	4
Height (in.)	54	36	24	16

Consecutive terms have a common ratio of $\frac{2}{3}$. You can write a model as an exponential function or as a geometric sequence.

Exponential function: $f(x) = 81\left(\frac{2}{3}\right)^x$, where x is the bounce number

Geometric sequence: $a_1 = 54$, $a_n = \frac{2}{3}a_{n-1}$, where n is the bounce number

CA CC F-LE.3

Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Key Vocabulary

linear function *(función lineal)*
A function that can be written in the form $y = mx + b$, where x is the independent variable and m and b are real numbers. Its graph is a line.

quadratic function *(función cuadrática)*
A function that can be written in the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$.

exponential function *(función exponencial)*
A function of the form $f(x) = ab^x$, where a and b are real numbers with $a \neq 0$, $b > 0$, and $b \neq 1$.

What It Means to You

You will use tables to compare a linear function, a quadratic function, and an exponential function.

EXAMPLE F-LE.3

Compare the linear function $f(x) = 3x - 4$, the quadratic function $g(x) = 3x^3 - 4$, and the exponential function $h(x) = 3^x - 1000$. Find the smallest integer value for x where $h(x) > g(x) > f(x)$.

x	$f(x) = 3x - 4$	$g(x) = 3x^3 - 4$	$h(x) = 3^x - 1000$
1	$3(1) - 4 = -1$	$3(1)^3 - 4 = -1$	$3^1 - 1000 = -997$
2	$3(2) - 4 = 2$	$3(2)^3 - 4 = 20$	$3^2 - 1000 = -991$
3	$3(3) - 4 = 5$	$3(3)^3 - 4 = 77$	$3^3 - 1000 = -973$
4	$3(4) - 4 = 8$	$3(4)^3 - 4 = 188$	$3^4 - 1000 = -919$
5	$3(5) - 4 = 11$	$3(5)^3 - 4 = 371$	$3^5 - 1000 = -757$
6	$3(6) - 4 = 14$	$3(6)^3 - 4 = 644$	$3^6 - 1000 = -271$
7	$3(7) - 4 = 17$	$3(7)^3 - 4 = 1025$	$3^7 - 1000 = 1187$
8	$3(8) - 4 = 20$	$3(8)^3 - 4 = 1532$	$3^8 - 1000 = 5561$
9	$3(9) - 4 = 23$	$3(9)^3 - 4 = 2183$	$3^9 - 1000 = 18{,}683$

The smallest integer value where $h(x) > g(x) > f(x)$ is 7. When $x \geq 7$, the exponential function has the greatest value of all three functions.

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CA CC F-LE.5

Interpret the parameters in a linear or exponential function in terms of a context.

Key Vocabulary

linear function *(función lineal)*
A function that can be written in the form $y = mx + b$, where x is the independent variable and m and b are real numbers. Its graph is a line.

parameter *(parámetro)*
One of the constants in a function or equation that may be changed. Also the third variable in a set of parametric equations.

What It Means to You

You will write a new function based on changes to the parameters of the function.

EXAMPLE F-LE.5

A company charges an initial fee of \$150 plus \$30 per hour to rent a limousine. The total cost of a limousine ride that lasts h hours is represented by the function $L(h) = 30h + 150$. For each change to a parameter, write a new function.

A. The company raises the initial fee from \$150 to \$175.

The change to the initial fee affects the y-intercept, so the new function is $L(h) = 30h + 175$.

B. The company lowers the hourly rate from \$30 to \$24.

The change to the hourly rate affects the slope, so the new function is $L(h) = 24h + 150$.

CA CC G-CO.1

Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Key Vocabulary

point *(punto)*
An undefined term in geometry, a point is a specific location and has no size.

line *(línea)*
An undefined term in geometry, a line is a straight path that has no thickness and extends forever.

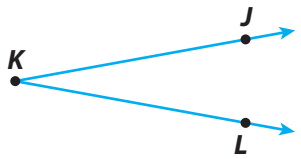
plane *(plano)*
An undefined term in geometry, a plane is a flat surface that has no thickness and extends forever.

What It Means to You

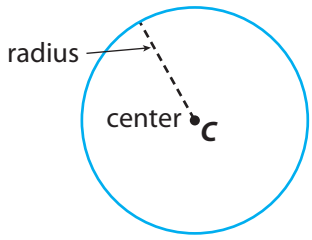
You will use the undefined terms *point*, *line*, and *plane* to write definitions for geometric terms such as *line segment* and *ray*.

EXAMPLE G-CO.1

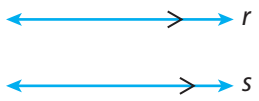
$\angle JKL$ is formed by two rays, $\overrightarrow{KJ}$ and $\overrightarrow{KL}$, that have a common endpoint K.



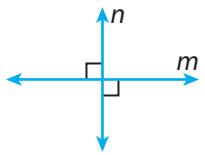
A circle is the set of all points in a plane that are the same distance from a point called the center of the circle. A line segment drawn from the circle to the center is called the radius of the circle.



Lines r and s are parallel lines because they lie in the same plane and never intersect.



Lines m and n are perpendicular lines because they intersect at 90° angles.



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CA CC G-CO.2

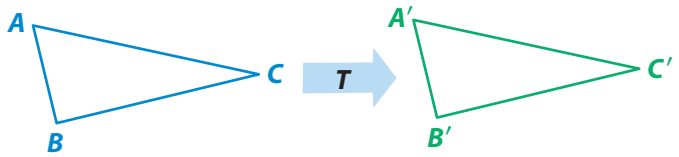
Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

What It Means to You

You will learn to think of a transformation as a function.

EXAMPLE G-CO.2

Use function notation to describe the translation that maps the vertices of triangle ABC to the vertices of triangle $A'B'C'$.



Function notation:

$$T(A) = A',\ T(B) = B',\ \text{and}\ T(C) = C'$$

CA CC G-CO.3

Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

Key Vocabulary

line symmetry *(simetría de línea)*
A figure that can be reflected across a line so that the image coincides with the preimage has line symmetry.

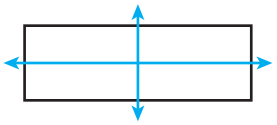
rotational symmetry *(simetría de rotación)*
A figure that can be rotated about a point by an angle less than 360° so that the image coincides with the preimage has rotational symmetry.

What It Means to You

You will use rotations and reflections to show that figures can have line symmetry, rotational symmetry, or both.

EXAMPLE G-CO.3

The rectangle shown has line symmetry. The horizontal line and the vertical line are its lines of symmetry.



If the top half is reflected across the horizontal line of symmetry, the top half coincides with the bottom half.

If the left half is reflected across the vertical line of symmetry, the left half coincides with the right half.

The rectangle also has rotational symmetry. If it is rotated 180° around the point of intersection of the two lines of symmetry, the rectangle coincides with itself.

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CA CC G-CO.4

Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Key Vocabulary

rotation *(rotación)*
A transformation that turns a figure by a specified angle about a point called the center of rotation.

reflection *(reflexión)*
A transformation that flips a figure across a line, called the line of reflection, to create a mirror image.

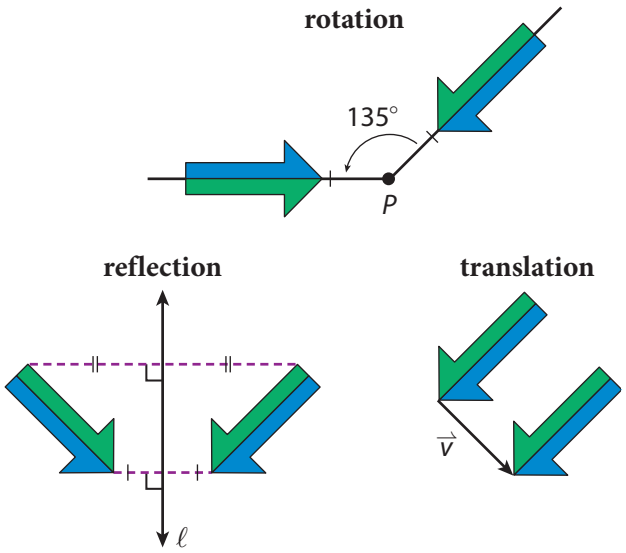
translation *(traslación)*
A transformation that slides every point of a figure along a given vector.

What It Means to You

You will learn to define rotations, reflections, and translations using precise geometric language.

EXAMPLE G-CO.4

These images illustrate the key features of each transformation.



CA CC G-CO.5

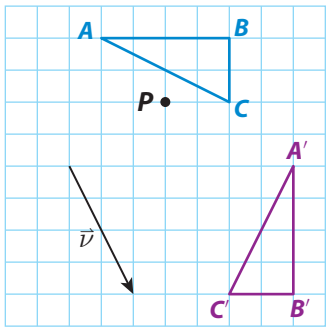
Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

What It Means to You

You will draw the image of a figure after a sequence of transformations.

EXAMPLE G-CO.5

$\triangle A'B'C'$ is the image of $\triangle ABC$ after a 270° counterclockwise rotation around point P followed by a translation along $\vec{v}$.



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CA CC G-CO.6

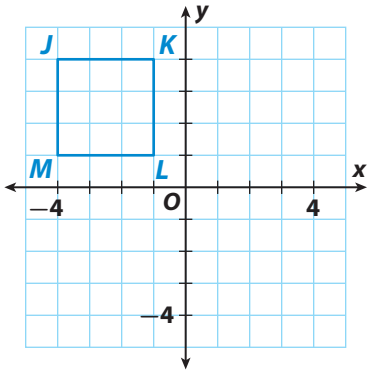
Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

What It Means to You

You will describe the effect of performing translations, reflections, and rotations on a figure.

EXAMPLE G-CO.6

Square *JKLM* is reflected across the *y*-axis, rotated 180° about *O*, and translated along the vector $\langle -2, -1 \rangle$.



The reflection maps the square from Quadrant II to Quadrant I. The rotation then maps the square from Quadrant I to Quadrant III. Finally, the translation moves the square 2 units left and 1 unit down.

The final image of *JKLM* is a square in Quadrant III that is farther from the axes than the original square.

CA CC G-CO.7

Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Key Vocabulary

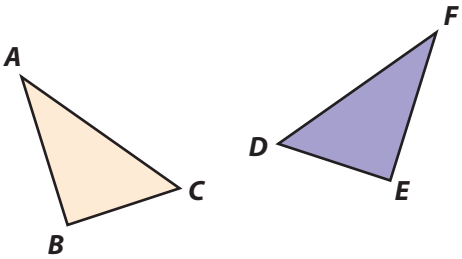
congruent *(congruente)*
Two plane figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations). The symbol for congruent is ≅.

What It Means to You

You will learn to use rigid motions to show that triangles are congruent.

EXAMPLE G-CO.7

Two pieces of colored glass have corresponding pairs of angles and sides that are congruent. How can you show that the triangles are congruent?



Use a translation to map point *C* to point *D*. Then, because corresponding pairs of sides and angles are congruent, you can reflect $\triangle ABC$ across a line through *C* and *D* so that point *A* maps to point *F* and point *B* maps to point *E*. The triangles are congruent because a sequence of rigid motions (translation, reflection) maps one triangle onto the other.

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CA CC G-CO.8

Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Key Vocabulary

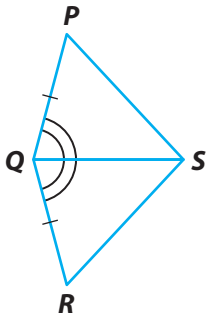
theorem *(teorema)*
A conjecture that can be proved using a logical sequence of definitions, postulates, and already-proven theorems.

What It Means to You

You will use rigid motions to justify some minimum criteria for deciding whether two triangles are congruent.

EXAMPLE G-CO.8

How can you show that $\triangle PQS$ and $\triangle RQS$ are congruent?



Because $\overline{QS} \cong \overline{QS}$, $\angle PQS \cong \angle RQS$, and $\overline{PQ} \cong \overline{RQ}$, you can reflect $\triangle PQS$ across $\overline{QS}$ so that it maps onto $\triangle RQS$. Knowing that two pairs of corresponding sides are congruent and their included angles are congruent is enough to show that the triangles are congruent. This is the basis of the SAS Triangle Congruence Theorem.

CA CC G-CO.12

Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

Key Vocabulary

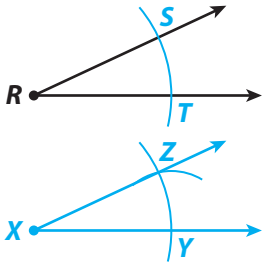
construction *(construcción)*
A method of creating a figure that is said to be geometrically precise. Figures may be constructed by using a compass and straightedge, geometry software, or paper folding.

What It Means to You

You will learn to draw precise geometric figures without using a protractor or a ruler.

EXAMPLE G-CO.12

Pete is designing a skateboard ramp with an incline angle of $\angle R$. How can he construct a copy of $\angle R$ using a compass and a straightedge?



- Use a straightedge to draw a ray with endpoint X.
- Draw an arc centered at R, labeling the intersections S and T.
- With the same compass width, draw an arc centered at X and intersecting the ray. Label the intersection Y.
- Using compass width ST, draw an arc centered at Y and intersecting the previous arc. Label the intersection Z.
- Use a straightedge to draw a ray from X through Z.

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CA CC **G-CO.13**

Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Key Vocabulary

inscribed polygon *(polígono inscrito)*
A polygon in which every vertex of the polygon lies on the circle.

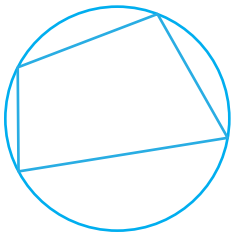
What It Means to You

You will learn to construct polygons inscribed in circles.

EXAMPLE G-CO.13

A polygon is inscribed in a circle if all of the polygon's vertices lie on the circle.

In the figure, the quadrilateral is inscribed in the circle, because all 4 vertices lie on the circle.



Every regular polygon can be inscribed in a circle. You can draw some of them using geometric constructions.

CA CC **G-GPE.4**

Use coordinates to prove simple geometric theorems algebraically.

Key Vocabulary

coordinate *(coordenada)*
A number used to identify the location of a point. On a number line, one coordinate is used. On a coordinate plane, two coordinates are used, called the *x*-coordinate and the *y*-coordinate.

What It Means to You

You will use slope criteria for parallel and perpendicular lines to analyze figures in the coordinate plane.

EXAMPLE G-GPE.4

$\triangle PQR$ has vertices $P(-3, -2)$, $Q(-2, 2)$, and $R(2, 1)$. Prove that $\triangle PQR$ is a right triangle.

Find the slopes of the segments that form the sides of the triangle. If the slopes of any two sides have a product of -1, then the sides are perpendicular and form a right angle.

$$\text{Slope of } \overline{PQ} = \frac{2-(-2)}{-2-(-3)} = 4$$

$$\text{Slope of } \overline{QR} = \frac{1-2}{2-(-2)} = -\frac{1}{4}$$

$$(\text{Slope of } \overline{PQ}) \cdot (\text{Slope of } \overline{QR}) = 4 \cdot \left(-\frac{1}{4}\right) = -1$$

Because $\overline{PQ}$ and $\overline{QR}$ are perpendicular and form a right angle, $\triangle PQR$ is a right triangle.

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CA CC G-GPE.5

Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Key Vocabulary

parallel lines *(líneas paralelas)*
Lines in the same plane that do not intersect. Parallel lines have the same slope.

perpendicular lines *(líneas perpendiculares)*
Lines that intersect to form right angles. If two lines are perpendicular, the product of their slopes is -1.

What It Means to You

You will learn to solve problems involving parallel and perpendicular lines by using slope relationships.

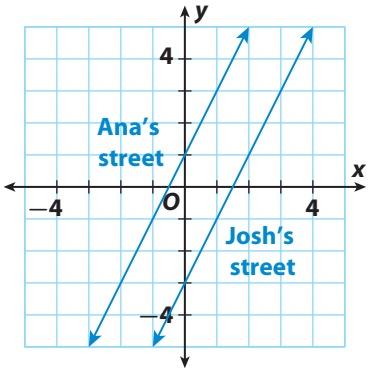
EXAMPLE G-GPE.5

Ana says that the street she lives on is parallel to the street Josh lives on. Is Ana's statement correct? Explain your reasoning.

Ana's street: $-2x + y = 1$

Josh's street: $\frac{1}{2}y + \frac{3}{2} = x$

Yes; writing each equation in slope-intercept form gives $y = 2x + 1$ for Ana's street and $y = 2x - 3$ for Josh's street. The slopes are the same, so the streets are parallel. You can check Ana's statement by graphing the two lines.



CA CC G-GPE.7

Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Key Vocabulary

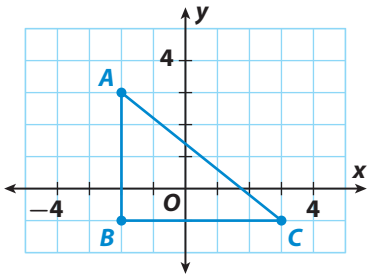
distance formula *(fórmula de distancia)*
In a coordinate plane, the distance d from (x_1, y_1) to (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

What It Means to You

You will learn to use the distance formula to find perimeters and areas.

EXAMPLE G-GPE.7

Find the perimeter of the triangle.



Find the length of each side. Then add.

$\overline{AB}$ is vertical, so $AB = 4$ units.

$\overline{BC}$ is horizontal, so $BC = 5$ units.

Use the distance formula to find AC.

$$AC = \sqrt{(3 - (-2))^2 + ((-1) - 3)^2} = \sqrt{25 + 16} = \sqrt{41}$$

$$AB + BC + AC = 4 + 5 + \sqrt{41} \text{ units.}$$

Use a calculator to evaluate the expression and round to the nearest tenth. So, the perimeter is about 15.4 units.

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CA CC **S-ID.1**

Represent data with plots on the real number line (dot plots, histograms, and box plots).

Key Vocabulary

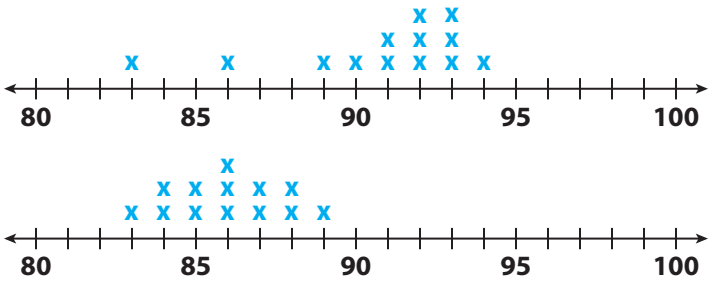
dot plot *(diagrama de puntos)*
A data representation that uses a number line and x's or dots to show frequency.

What It Means to You

You can represent data sets using various models and use those models to interpret the information.

EXAMPLE S-ID.1

Class Scores on First Test (top) and Second Test (bottom)



How do the medians of the two sets of test scores compare?

Look at each dot plot and locate the median. The median for the first test (92) is higher than the median for the second test (86).

CA CC **S-ID.2**

Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

Key Vocabulary

mean *(media)*
The average of the data values.

median *(mediana)*
The middle value when values are listed in numerical order.

interquartile range *(rango intercuartil)*
A measure of the spread of a data set, obtained by subtracting the first quartile from the third quartile.

What It Means to You

You can use the mean and median of data sets to compare the centers of the data sets. You can use the range, interquartile range, or standard deviation to compare the spreads of the data sets.

EXAMPLE S-ID.2

The lengths in feet of the alligators at a zoo are 9, 7, 12, 6, and 10. The lengths in feet of the crocodiles at the zoo are 13, 10, 8, 19, 18, and 16.

What is the difference between the mean length of the crocodiles and the mean length of the alligators?

Alligators: $\frac{9 + 7 + 12 + 6 + 10}{5} = 8.8$

Crocodiles: $\frac{13 + 10 + 8 + 19 + 18 + 16}{6} = 14$

$14 - 8.8 = 5.2$ ft

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CA CC S-ID.3

Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Key Vocabulary

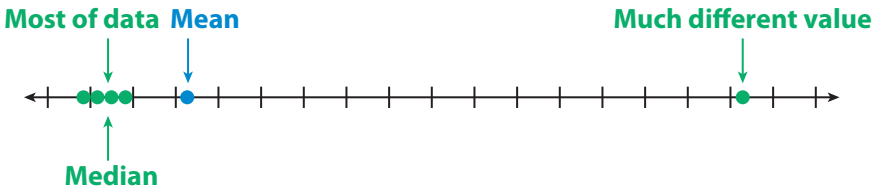
outlier *(valor extremo)*
A data value that is far removed from the rest of the data.

What It Means to You

Always examine the displays and statistics for a data set in its own particular context so that you can draw valid conclusions.

EXAMPLE S-ID.3

Outliers



The outlier pulls the mean to the right. The median better represents a "typical" value.

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CA CC S-ID.5

Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

Key Vocabulary

frequency table *(tabla de frequencia)*
A table that lists the number of times, or frequency, that each data value occurs.

joint relative frequency *(frequencia relative conjunta)*
A relative frequency that is found by dividing a frequency that is not in the Total row or the Total column by the grand total.

marginal relative frequency *(frequencia relative marginal)*
A relative frequency that is found by dividing a row total or column total by the grand total.

conditional relative frequency *(frequencia relative condicional)*
A relative frequency that is found by dividing a frequency that is not in the Total row or the Total column by the frequency's row total or column total.

What It Means to You

Two-way frequency tables give you a visual way to organize data categorized by two different variables so that you can more easily identify relationships.

EXAMPLE S-ID.5

The two-way frequency table below shows the numbers of households in a study that own a dog, a cat, or both.

	Owns a Cat		
Owns a Dog	**Yes**	**No**	**Total**
Yes	15	24	39
No	18	43	61
Total	33	67	100

Here are a few conclusions you can draw from the table.

- $\frac{39}{100}$, or 39%, of households own a dog; $\frac{33}{100}$, or 33%, own a cat.
- $\frac{15}{100}$, or 15%, own a dog and a cat; $\frac{43}{100}$, or 43%, own neither.
- Of dog owners, $\frac{15}{39}$, or about 38.5%, also own a cat.
- Of cat owners, $\frac{15}{33}$, or about 45.5%, also own a dog.

You can also show the data in a two-way *relative* frequency table by dividing each number by the grand total and expressing the answer as a percent.

joint relative frequency

	Owns a Cat		
Owns a Dog	**Yes**	**No**	**Total**
Yes	15%	24%	39%
No	18%	43%	61%
Total	33%	67%	100%

This cell should always be 100% in a two-way relative frequency table.

marginal relative frequencies

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CA CC S-ID.6a

Fit a function to the data; use functions fitted to data to solve problems in the context of the data.

Key Vocabulary

function *(función)*
An input-output relationship that has exactly one output for each input

What It Means to You

You can use a function to approximate the relationship between two variables.

EXAMPLE S-ID.6a

This table shows the number of Blu-ray discs Clarissa has sold each year since she opened her video store in 2006.

Years since 2006	0	1	2	3	4	5	6
Number of Blu-rays sold	27	117	252	313	395	423	573

Enter the data from the table on a graphing calculator. Use the calculator's linear regression feature to find the model of the data. This data can be represented by the linear function $f(x) = 85.46x + 43.61$.

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CA CC S-ID.6b

Informally assess the fit of a function by plotting and analyzing residuals.

Key Vocabulary

residual *(residuo)*
The signed vertical distance between a data point and a line of fit.

line of best fit *(línea de mejor ajuste)*
The line that comes closest to all of the points in a data set.

What It Means to You

You will determine the line of best fit by comparing the residuals for two lines.

EXAMPLE S-ID.6b

The data in the table can be represented by the line $y = 0.3x + 16$ or the line $y = 0.4x + 12$. Determine which line is the better fit.

x	0	25	50	75	100
y	13.8	21.4	29.8	38.1	47.7

STEP 1 Find the residuals for each line.

y predicted by $y = 0.3x + 16$	Residual for $y = 0.3x + 16$	y predicted by $y = 0.4x + 12$	Residual for $y = 0.4x + 12$
16	−2.2	12	1.8
23.5	−2.1	18	3.4
31	−2.1	28	1.8
38.5	−0.4	38	0.1
46	1.7	48	−0.3

STEP 2 Square the residuals and find their sum.

$$(-2.2)^2 + (-2.1)^2 + (-2.1)^2 + (-0.4)^2 + (1.7)^2 = 16.71$$

$$(1.8)^2 + (3.4)^2 + (1.8)^2 + (0.1)^2 + (-0.3)^2 = 18.14$$

The sum of the squares for $y = 0.3x + 16$ is smaller, so that line is a better fit for the data.

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CA CC S-ID.6c

Fit a linear function for a scatter plot that suggests a linear association.

Key Vocabulary

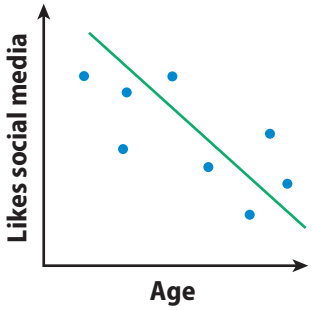
linear function *(función lineal)*
A function whose graph is a straight line.

What It Means to You

In a linear relationship one quantity is directly proportional to another. For example, the greater the altitude, the thinner the air is in the atmosphere.

EXAMPLE S-ID.6c

Reed drew a line in his scatter plot, suggesting a linear correlation between age and how much people like social media.



CA CC S-ID.7

Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Key Vocabulary

slope *(pendiente)*
A measure of the steepness of a line. If (x_1, y_1) and (x_2, y_2) are any two points on the line, the slope of the line, known as m, is represented by the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$.

***y*-intercept** *(intersección con el eje y)*
The y-coordinate(s) of the point(s) where a graph intersects the y-axis.

What It Means to You

You will determine the meaning of the slope and the intercept for a line of best fit.

EXAMPLE S-ID.7

A scientist is performing an experiment. The table shows the number of bacteria on a slide x minutes after the start of an experiment. The equation of the line of best fit is $y = -1.8x + 95$. Determine what the slope and the y-intercept of the line represent.

x	0	5	10	15	20
y	97	85	75	69	57

The slope of the line is -1.8. This represents the rate at which the number of bacteria on the slide is decreasing.

The y-intercept of the line is 95. This represents the approximate number of bacteria at the start of the experiment.

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CA CC S-ID.8

Compute (using technology) and interpret the correlation coefficient of a linear fit.

Key Vocabulary

correlation coefficient *(coeficiente de correlación)*
A number r, where $-1 \leq r \leq 1$, that describes how closely the points in a scatter plot cluster around the least-squares line.

What It Means to You

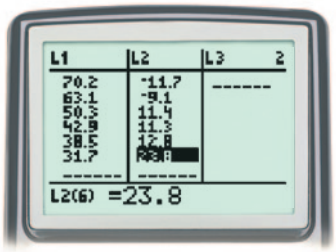
You will use a calculator to find the correlation coefficient for a line of best fit.

EXAMPLE S-ID.8

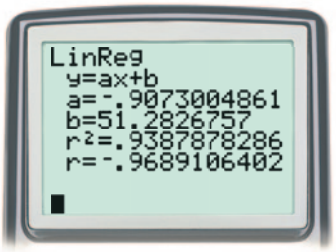
The table shows the latitude and average temperatures for several cities. Use a calculator to find correlation coefficient, r, for a line of best fit.

Latitude	70.2° N	63.1° N	50.3° N	42.9° N	38.5° N	31.7° N
Temperature (°C)	−11.7	−9.1	11.4	11.3	12.8	23.8

STEP 1 Press the STAT key and select 1:Edit to enter the latitudes in column L1 and the average temperatures in column L2.



STEP 2 Press the STAT key and choose CALC. From the menu, choose 4:LinReg(ax+b). The calculator will display the correlation coefficient, r, for the line of best fit. Since the correlation coefficient is close to 1, the line of fit has a strong positive correlation.



CA CC S-ID.9

Distinguish between correlation and causation.

Key Vocabulary

correlation *(correlación)*
A measure of the strength and direction of the relationship between two variables or data sets.

What It Means to You

You will determine whether a given variable is the cause of a positive correlation.

EXAMPLE S-ID.9

A survey of teachers of grades 1–6 found that students with larger shoe sizes also have higher scores on tests covering multiplication facts. Determine the correlation, and discuss whether the correlation is a result of causation.

Larger shoe sizes correspond to higher scores on multiplication tests, so there is a positive correlation between shoe size and test scores. It is unlikely that larger feet cause higher scores or vice versa. It is more likely that a lurking variable, such as age, causes students to both have larger shoe sizes and have higher multiplication scores. Older students have had more time to learn multiplication facts, so age could be the cause of the positive correlation.

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